Horizons, Thermofield Dynamics and Quantum Information

Sang Pyo Kim (Kunsan Nat'l Univ. & APCTP)

in collaboration with Xian-Hui Ge (APCTP)

ICGA8, Nara, Japan, August 29-September 1, 2007

Outline

- Motivation
- Horizons of Rindler Spacetime and Black Holes
- Thermofield Dynamics
- Quantum Information
- Summary

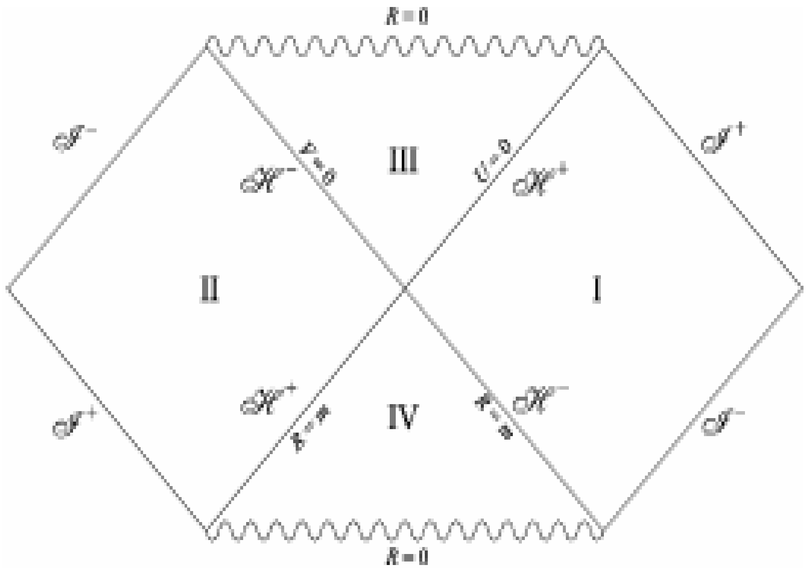
Motivation

- There are many reasons to revive the interest on black holes.
- The self-gravitation (back reaction) of emission from black hole has been calculated and a non-thermal correction to the Hawking radiation was found [Kraus & Wilczek (‘95); Keski-Vakkuri & Kraus (‘97); Parikh & Wilczek (‘00)].
- The Hawking radiation has been reinterpreted as quantum tunneling of field modes across the horizon.

Motivation

- Higher dimensional Kerr-(NUT)-AdS solutions have been found [Chen & Cvetic & Gibbons & Lu & Page & Pope].
- Symmetry and separation of fields in higher dimensional black holes [Frolov & Krtous & Kubiznak & Page].
- Quantum information in non-inertial frames has attracted attention [Alsing & Milburn (’03); Fuentes-Schuller & Mann (‘05), (‘06)].
- Quantum information in higher dimensional black hole spacetimes has been considered [X-H. Ge & SPK, arXiv:0707.4523].

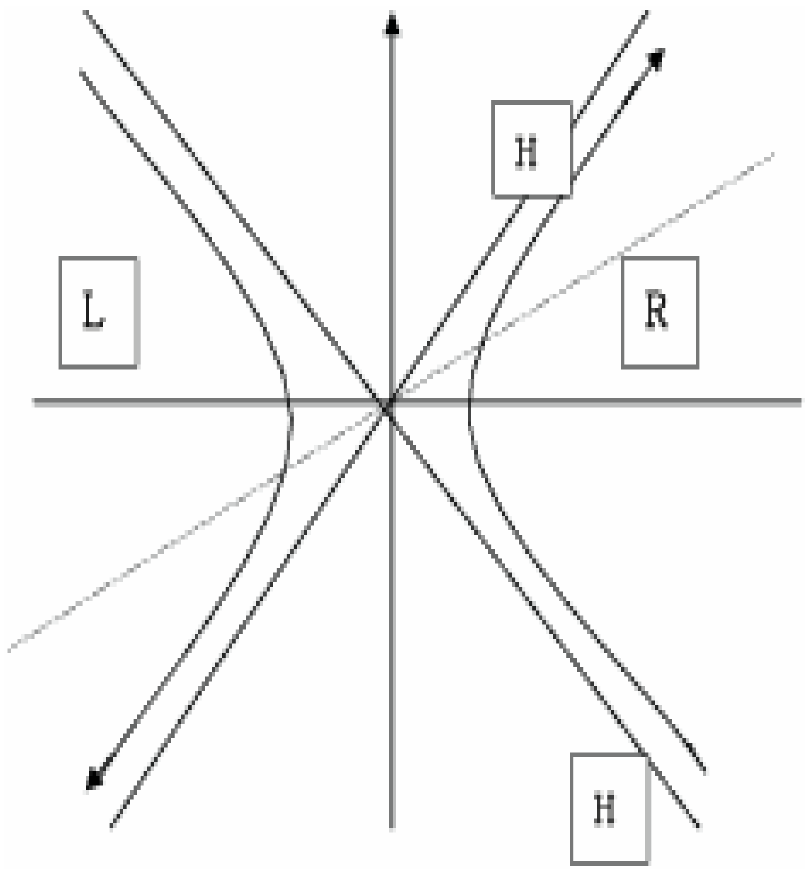
Hawking Radiation



Penrose diagram for black hole

- Hawking discovered a thermal radiation from black hole [Hawking , CMP 43 (‘75)]:
- Israel interpreted the Hawking radiation using the thermofield dynamics [Israel, PLA 57 (’76)].

Horizon of Rindler Spacetime



R is causally connected to an outside observer

- uniform acceleration a
- R (right wedge):

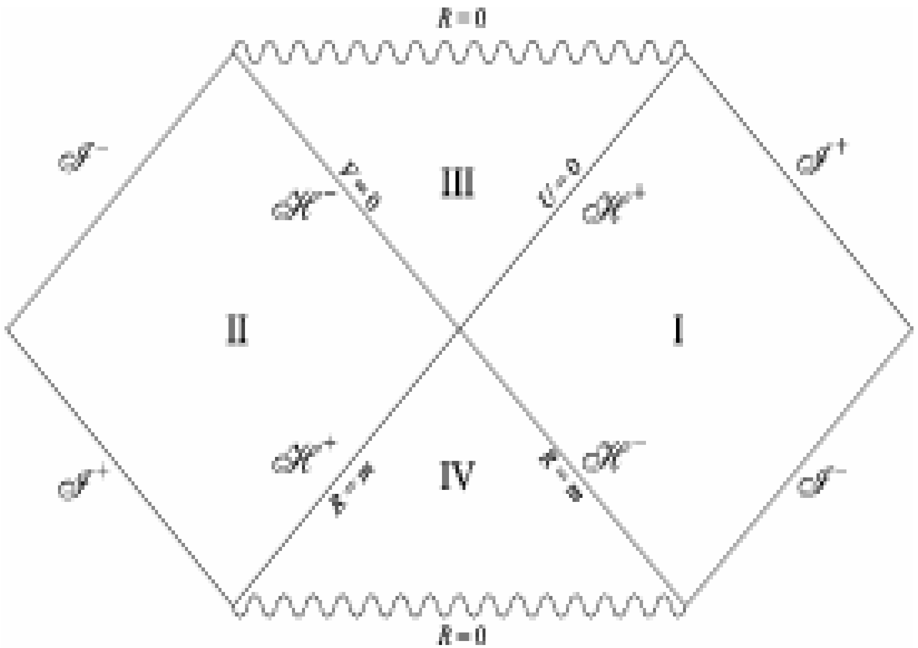
  $at = e^{a\zeta} \sinh(a\tau)$

  $az = e^{a\zeta} \cosh(a\tau)$
- L (left wedge)

  $at = -e^{a\zeta} \sinh(a\tau)$

  $az = -e^{a\zeta} \cosh(a\tau)$
- The quantization in the Rindler coordinates differs from the Minkowski quantization due to the presence of the horizon H.

Horizon of a Black Hole



- Near the horizon the Schwarzschild black hole

$ds^2 = -(1 - 2M/r)dt^2 + (1 - 2M/r)^{-1} dr^2 + r^2 d\Omega^2$

approximately has the Rindler metric

$ds^2 = -(\kappa x)^2 dt^2 + dx^2 + (1/4\kappa^2) d\Omega^2$,

$r - 2M = \kappa x^2/2$,

$\kappa = 1/(4M)$, surface gravity

Thermofield Dynamics (I)

- TFD was introduced by Takahashi & Umezawa [Collect. Phenom. 2 (1975)]
- The essential idea of TFD is to describe thermal states of a quantum field by the zero-temperature field theory by doubling the degrees of freedom (the system plus a fictitious system):

$$\mathbf{H} = H_0 - \underline{H}_0$$

- The role of the fictitious system $\underline{H}_0$ is just to double the degrees of freedom without any interaction.
- The Hamiltonian for a boson field now becomes

$$\mathbf{H} = \sum \omega_k \left[a^+_k a_k - \underline{a}^+_k \underline{a}_k\right], \quad [a_k, \underline{a}^+_k] = 0$$

Thermofield Dynamics (II)

- The vacuum of the boson field: $a_k |0\rangle = 0$
- The particle states (Fock space) are given by

$$a^{+}_{k1}\, a^{+}_{k2} \cdots a^{+}_{kn} |0\rangle = |k_1, k_2, \cdots, k_n\rangle$$

- The fictitious system also has the Fock space:

$$\underline{a}_k |\underline{0}\rangle = 0$$

$$\underline{a}^{+}_{k1}\, \underline{a}^{+}_{k2} \cdots \underline{a}^{+}_{kn} |\underline{0}\rangle = |\underline{k}_1, \underline{k}_2, \cdots, \underline{k}_n\rangle$$

- The vacuum state and the Fock space of the total system **H** are

$$|0\rangle\rangle = |0\rangle \otimes |\underline{0}\rangle$$

$$\boldsymbol{H} = H \otimes \underline{H}$$

Thermofield Dynamics (III)

- For each mode k, the thermal vacuum is given by

  $|0, \beta\rangle = (1- e^{-\omega k\,\beta})^{1/2} \sum_n e^{-n\omega k\,\beta/2}\,|n_k\rangle \otimes |\underline{n}_k\rangle$

- Define the thermal annihilation/ creation operator through the Bogoliubov tranformation

  $a_k(\beta) = \cosh\theta(\beta)\, a_k - \sinh\theta(\beta)\, \underline{a}^+_k = U(\theta)\, a_k\, U^+(\theta)$

  with $\tanh\theta(\beta) = e^{-\omega k\,\beta/2}$, $U(\theta) = e^{-\theta(\beta)(ak\,\underline{a}k - a+k\,\underline{a}+k)}$

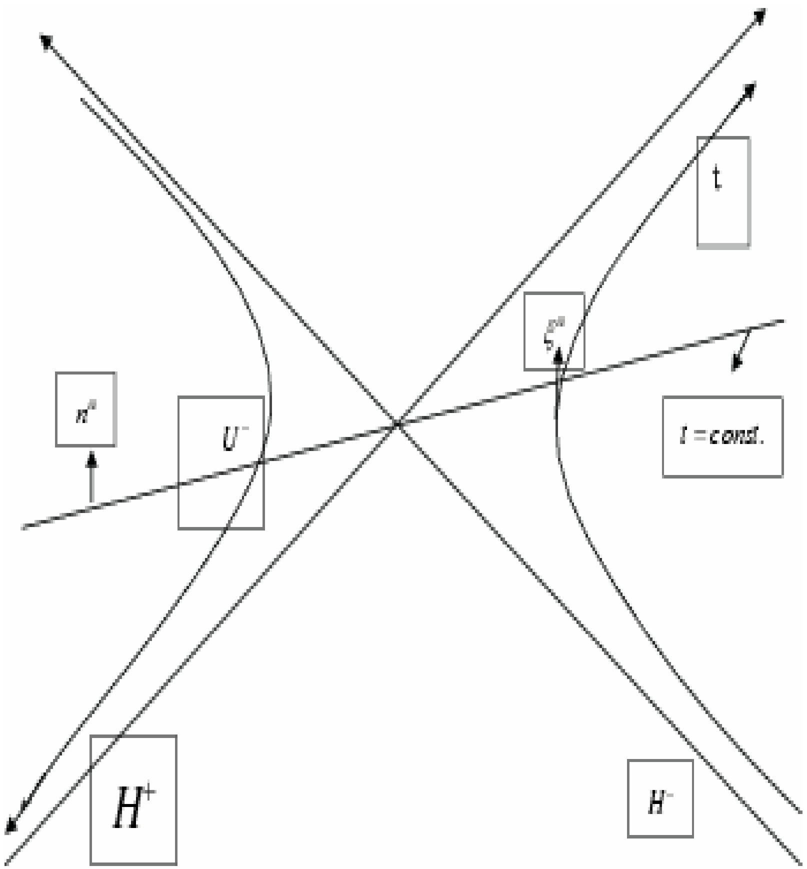
- Then the thermal vacuum is the vacuum of the thermal annihilation operator or the two-mode squeezed vacuum state

  $a_k(\beta)\,|0, \beta\rangle = 0$ or $|0, \beta\rangle = U(\theta)\,|0\rangle\rangle$

- The expectation values of operators in the thermal state are simply given by

  $\langle\hat{O}\rangle_T = \langle 0, \beta|\,\hat{O}\,|0, \beta\rangle$

Thermofield Dynamics of Rindler and Black Holes (I)



Extended Kruskal manifold

- The Unruh temperature for an accelerated observer

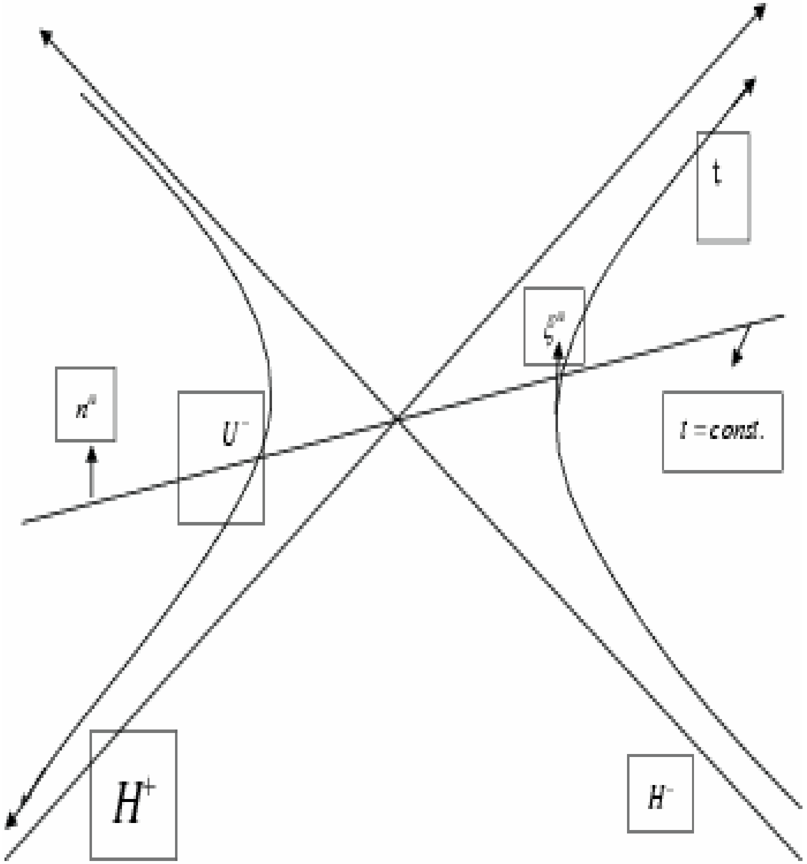
$$T_R = a/(2\pi)$$

- The Hawking temperature

$$T_H = \kappa/(2\pi)$$

- The surface gravity of a black hole is the acceleration of a static particle near the horizon seen by an infinite observer.

Thermofield Dynamics of Rindler and Black Holes (II)



Extended Kruskal manifold

- Use a Killing vector to define the positive frequency mode solutions of the Klein-Gordon (boson) equation.
- ∂_t is a timelike Killing vector in (I) and ∂_{-t} in (II).
- The Kruskal coordinates $U = (-/+)(1/\kappa)e^{-\kappa(t-r^*)}$, $V = (+/-)(1/\kappa)e^{\kappa(t+r^*)}$ provide another timelike Killing vector $\partial/\partial U$.

Thermofield Dynamics of Rindler and Black Holes (III)

- The Bogoliubov transformations between the Kruskal operators d_k and the Schwarzchild operators b_k are [Unruh (76)]

$b_k^{I} = [2 \sinh(\omega_k \beta/2)]^{-1/2} (e^{\omega k\beta/2} d_k^{I} + e^{-\omega k\beta/2} d_k^{+II})$

$b_k^{II} = [2 \sinh(\omega_k \beta/2)]^{-1/2} (e^{\omega k\beta/2} d_k^{II} + e^{-\omega k\beta/2} d_k^{+I})$

- In terms of the two-squeeze operator, the inverse transformations are given by

$d_k^{\sigma} = T_k(r)\, b_k^{\sigma}\, T_k^{+}(r)$

$T_k(r) = e^{-r\,(b^{I} b^{II} - b^{+I} b^{+II})}$

$\tanh r = e^{-\omega k\beta/2}$, $(\sigma = I, II)$

Quantum Information

- The Kruskal vacuum $d_k^\sigma |0\rangle\rangle_M = 0, (\sigma = I, II)$ is now given by the two-mode squeezed state:

$$|0\rangle\rangle_{\mathbf{M}} = \prod_k T_k(r) |0\rangle_I \otimes |\underline{0}\rangle_{II}$$
$$= \prod_k (1/\cosh r)\, e^{\tanh r\, (b^+{}_I b^+{}_{II})} |0\rangle_I \otimes |\underline{0}\rangle_{II}$$

- The excited states are

$$|n\rangle\rangle_{\mathbf{M}} = \prod_k (d_k^I)^{nk}/(n_k!)^{1/2} |0\rangle\rangle_{\mathbf{M}}$$
$$= \prod_k T_k(r) |n\rangle_I \otimes |\underline{0}\rangle_{II}$$

- The Fock space of quantum states in the black holes becomes equivalent to TFD and quantum information in black holes can be expressed by TFD.
- Caveat: TFD is originally defined for static systems (thermal equilibrium). To extend TFD to nonstationary spacetimes, we need time-dependent TFD (nonequilibrium) in the Liouville-von Neumann picture [SPK & Khanna (03)].

TFD in Curved Spacetimes

- Israel reinterpreted the Hawking radiation in TFD (‘76).
- The unitarily inequivalent Fock representations in curved spacetime [Martellini & Sodano & Vitiello (‘78)].
- Thermal states of Rindler, Schwarzchild black hole and de Sitter space [Laflamme (‘86)].
- Black hole entropy [Frolov & Novikov (‘93)].
- Nonequilibrium evolution of a quantum field in an expanding universe [SPK et al (‘96)].
- Vacuum structure of expanding geometry [Vitiello et al (‘00)].
- Nonequilibrium evolution of primordial gravitational waves [Koh & SPK (‘04)].
- Quantum entanglement entropy of black holes has been calculated [Iorio & Lambiase & Vitiello (‘04)].
- ……….

Summary

- The causally disconnected region of the extended Kruskal manifold is analogous to the fictitious system of thermofield dynamics.
- The Cauchy problem for a quantum field in a black hole or an accelerated frame has such a causally disconnected region.
- Thermofield dynamics may provide a very useful tool to study quantum states and quantum information outside the horizon and probably may help us understand the final state of black holes.