WZW-type action for NS-NS SFT

& its gauge fixing conditions

Hiroaki Matsunaga Univ. Tokyo

(with Keiyu Goto)

Komaba, Mar, 5th, 2015.

Introduction: Superstring Field Theory

Witten's Cubic Theory

Witten

Small-space theory (A_∞/L_∞-type actions)	Large-space theory (WZW-type actions)
NS open	NS open **Berkovits 95'**
NS closed	NS closed **Berkovits, Okawa, Zwiebach 04'**
NS-NS closed **Erler, Konopka, Sachs 14'**	NS-NS closed **H.M 14'**

Gauge fixing . . . OK

Gauge fixing . . . Not Yet

Introduction: Superstring Field Theory

Witten's Cubic Theory

Witten

Small-space theory (A_∞/L_∞-type actions)	Large-space theory (WZW-type actions)
NS open	NS open **Berkovits 95'**
NS closed	NS closed **Berkovits, Okawa, Zwiebach 04'**
NS-NS closed **Erler, Konopka, Sachs 14'**	NS-NS closed **H.M 14'**

↔

Gauge fixing . . . OK

Gauge fixing . . . Not Yet

Superstring field theory action

- difficulties -

Bosonic open String Fields

Vertex Op.		String Field	$\#_{gh}$
$\mathcal{V}(z) = c(z)e^{k\cdot X}$	$\rightarrow$	Ψ	1

Free Action : $S = \frac{1}{2}\langle\Psi, Q\Psi\rangle$ ($\#_{gh}$ of Q = 1)

<A, B > : BPZ inner product → ghost # anomaly !!

→ <A, B> = 0 except for ghost # of A + B = 3

→ Ghost # ~ Grading (like a differential form)

Bosonic open SFT Action

Witten's Cubic Action :

$$S = \frac{1}{2}\langle \Psi, Q\Psi \rangle + \frac{1}{3}\langle \Psi, \Psi * \Psi \rangle$$

EOM :

$$Q\Psi + \Psi^2 = 0$$

Gauge transf. :

$$\delta\Psi = Q\Lambda + [\![\Psi, \Lambda]\!]$$

Pure-gauge :

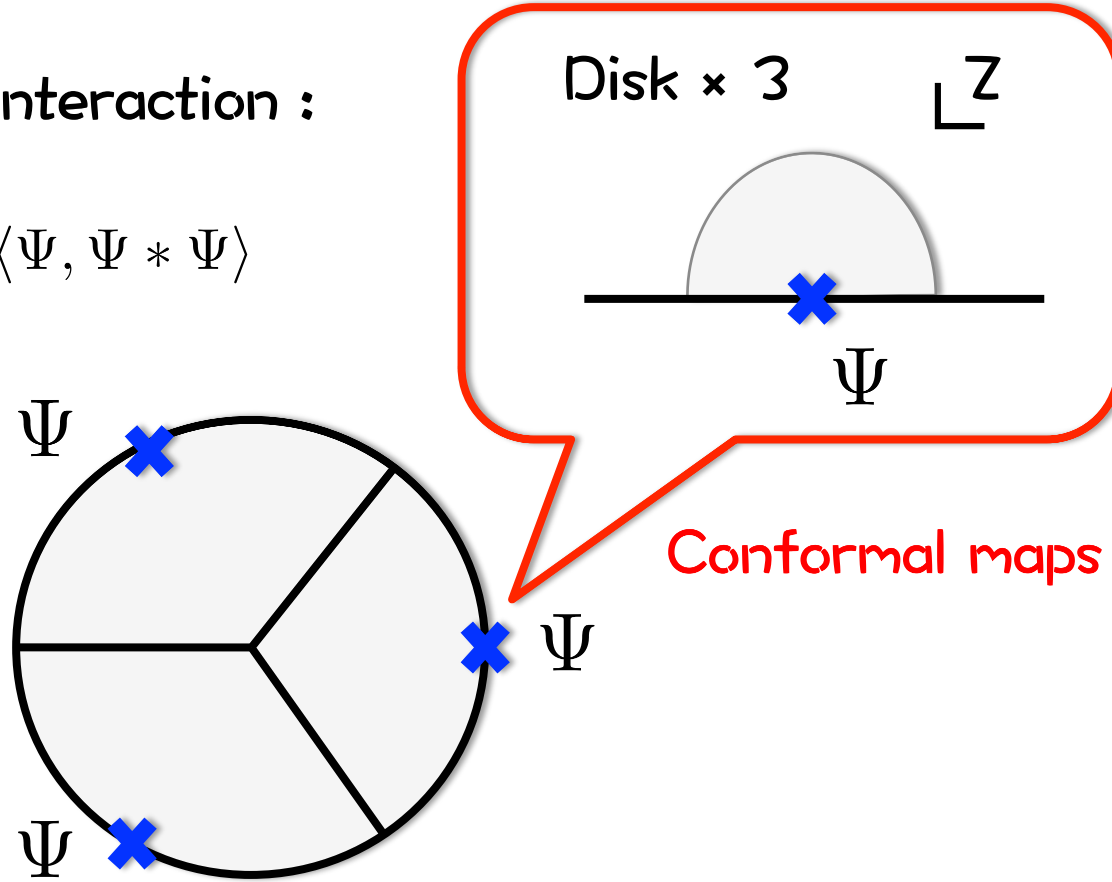
$$e^{-\Lambda} Q e^{\Lambda}$$

Interacting terms

Witten's Mid Point Interaction :

$$S = \frac{1}{2}\langle \Psi, Q\Psi \rangle + \frac{1}{3}\langle \Psi, \Psi * \Psi \rangle$$

$$\langle \Psi, \Psi * \Psi \rangle \quad =$$



→ Feynman Diagrams reproduce on-shell amp.

Note that $\#_{gh}$ of $\Psi = 1$ and $\#_{gh}$ of $Q = 1$

Superstring Fields

	Vertex Op.		String Field	($\#_{gh}$ | $\#_{pic}$)
Small :	$\mathcal{V}(z) = c(z)\delta(\gamma)V_m$	$\rightarrow$	Ψ	$(1\vert-1)$

In the "small" Hilbert space. . . ($\beta\gamma$-system)

<A, B > : BPZ product $\rightarrow$ ghost & picture # anomaly !!

$\rightarrow$ <A, B> = 0 except for ($\#_{gh}$ | $\#_{pic}$) of A + B = (3 | -2)

$\rightarrow$ Grading ~ Ghost & Picture # (like a super-form)

$\rightarrow$ Interacting term has problem !!

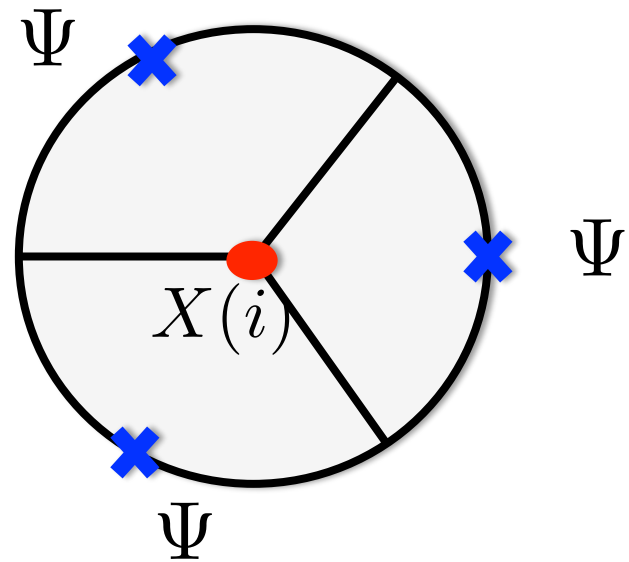
Attempt: Witten's Cubic Action

Witten's Cubic Action :

$$S = \frac{1}{2}\langle \Psi, Q\Psi \rangle + \frac{1}{3}\langle \Psi, X(i)\big(\Psi * \Psi\big)\rangle$$

$X(i)$: Picture Changing Operator (Mid Point !!)

$\langle \Psi, X(i)\big(\Psi * \Psi\big)\rangle$ =



This Product is Associative !!

Attempt: Witten's Cubic Action

- OPE of PCOs $X(i)$ is singular. . .

  ➡ Contact terms become DIVERGENT !!

- Broken gauge invariance. . .

⬇

Developments of technique :

Large-space actions (WZW-like actions)

Small-space actions (Regularized ver. of Witten theory)

Plan

0. Introduction
1. Review of NS theory
2. NS-NS Actions
3. Gauge fixing of WZW-like actions

1. Review of NS theory

- large-space -

(WZW-like action)

NS WZW-like Action

	Vertex Op.		String Field	($\#_{gh}$ | $\#_{pic}$)
Large :	$\mathcal{V}(z) = \xi(z) c e^{-\phi} V_m$	$\rightarrow$	Φ	$(0\|0)$

In the "large" Hilbert space. . . ($\eta\xi\phi$-system)

→ <A, B> = 0 except for ($\#_{gh}$ | $\#_{pic}$) of A + B = (2 | -1)

Note that "# of Q" = (1 | 0) and "# of η" = (1 | -1)

Free Action $$S = \frac{1}{2}\langle \eta\Phi, Q\Phi \rangle$$

NS WZW-like Action

Free Action	EOM	Gauge transf.
$S = \frac{1}{2}\langle \eta\Phi, Q\Phi \rangle$	$Q\eta\Phi = 0$	$\delta\Phi = Q\Lambda + \eta\Omega$

- Interacting terms . . . ??

Note that "# of ϕ" = (0 | 0) !

→ We can make a function of ϕ

without 'picture-changing problems'.

Berkovits' open Superstring Field Theory

Berkovits' WZW-like Action

$$S = \frac{1}{2}\langle e^{-\Phi}Qe^{\Phi}, e^{-\Phi}\eta e^{\Phi}\rangle + \frac{1}{2}\int_0^1 dt \langle e^{-t\Phi}\partial_t e^{t\Phi}, [\![e^{-t\Phi}Qe^{t\Phi}, e^{-t\Phi}\eta e^{t\Phi}]\!]\rangle$$

EOM : $\eta\left(e^{-\Phi}Qe^{\Phi}\right) = 0$

Gauge transf. : $e^{-\Phi}\delta e^{\Phi} = Q_{\mathcal{G}}\Lambda + \eta\Omega$

$$Q_{\mathcal{G}} := Q + [\![e^{-\Phi}(Qe^{\Phi}), \quad]\!]$$

→ (Formal) Pure-gauge is the key

For example, WZW model can be rewrite. . .

1. Familiar WZW action

$$S = \frac{1}{2g^2}\int d^2z \int_0^1 dt \ \mathrm{Tr}\Big(\partial_t (A_z\, A_{\bar{z}}) + A_t [A_z, A_{\bar{z}}] \Big)$$

$$A_i = e^{-\Phi} \left(\partial_i e^{\Phi} \right)$$

$$i = t, z, \bar{z}$$

Flatness conditions : $F_{ij} = \partial_i A_j - \partial_j A_i + [\, A_i,\ A_j \,] = 0$

$$S = \frac{1}{2g^2}\int d^2z \int_0^1 dt \ \mathrm{Tr}\Big((\partial_z A_t) A_{\bar{z}} + A_z (\partial_{\bar{z}} A_t) + A_t [A_z, A_{\bar{z}}] \Big)$$

← F_{tz} , $F_{t\bar{z}}$ = 0

$$= \frac{1}{2g^2}\int d^2z \int_0^1 dt \ \mathrm{Tr}\Big((\partial_z A_t) A_{\bar{z}} - (\partial_z A_{\bar{z}}) A_t \Big) = \frac{1}{g^2}\int d^2z \int_0^1 dt \ \mathrm{Tr}\big((\partial_z A_t) A_{\bar{z}} \big)$$

↑
$F_{z\bar{z}}$ = 0

2. WZW-like form (appears in SFT)

$$S = \frac{1}{g^2}\int d^2z \int_0^1 dt \ \mathrm{Tr}\big((\partial_z A_t)\, A_{\bar{z}} \big)$$

Rewriting Berkovits' Action

1. Familiar WZW form

$$S = -\frac{1}{2g^2}\int_0^1 dt\,\langle\!\langle\, \partial_t\,(A_\eta\, A_Q) + A_t\,\{A_Q\,,\,A_\eta\}\,\rangle\!\rangle$$

$$A_X := e^{-t\Phi}(Xe^{t\Phi})$$

$$X = Q, \eta, \delta, \partial_t$$

Conditions : $X A_Y - (-)^{XY} Y A_X + [\![A_X , A_Y]\!] = 0$

$$= \frac{1}{2}\langle\!\langle\, (\,\partial_t A_\eta + [\,A_t,\, A_\eta\,]\,)\, A_Q\,\rangle\!\rangle + \frac{1}{2}\langle\!\langle\, A_\eta\,(\partial_t A_Q)\,\rangle\!\rangle$$

$$= \frac{1}{2}\langle\!\langle\, (\eta A_t)\, A_Q\,\rangle\!\rangle + \frac{1}{2}\langle\!\langle\, A_\eta\,(Q' A_t)\,\rangle\!\rangle$$

$$= \frac{1}{2}\langle\!\langle\, (\eta A_t)\, A_Q\,\rangle\!\rangle - \frac{1}{2}\langle\!\langle\, (\eta A_Q)\, A_t\,\rangle\!\rangle = \langle\!\langle\, (\eta A_t)\, A_Q\,\rangle\!\rangle$$

2. WZW-like form (fundamental)

$$S = \int_0^1 \langle \eta A_{\partial_t}, A_Q\rangle = \int_0^1 dt \langle \eta\Phi, \mathcal{G}(t\Phi)\rangle$$

Pure-gauge

How to make 'large-space' NS action?

1. Consider a 'bosonic' pure-gauge solution.

$$\frac{\partial}{\partial t}\mathcal{G}(t) = Q_{\mathcal{G}(t)}\Lambda$$

2. Identify NS string fields with 'bosonic' gauge parameters.

$$\Phi = \Lambda$$

Ghost & picture #s match!!

Then, we obtain the 'large-space' NS action.

$$S = \int_0^1 dt \langle \eta\Phi, \mathcal{G}(t\Phi)\rangle$$

Heterotic String Field Theory

WZW-like Action

$$S = \int_0^1 dt \langle \eta V, \mathcal{G}(tV) \rangle$$

NS field V ⟷ Gauge para. Λ

EOM

$$\eta \mathcal{G}(V) = 0$$

Bosonic pure-gauge

$$\frac{\partial}{\partial t}\mathcal{G}(t\Lambda) = Q_{\mathcal{G}(t\Lambda)}\Lambda$$

$$Q_{\mathcal{G}} = Q + \sum_{n=1}^{\infty} \frac{\kappa^n}{n!}[\mathcal{G}^n, \quad]$$

Gauge transf.

(Function of δV) = $Q_{\mathcal{G}}\Lambda + \eta\Omega$

- small-space -

(Regularized ver. of Witten theory)

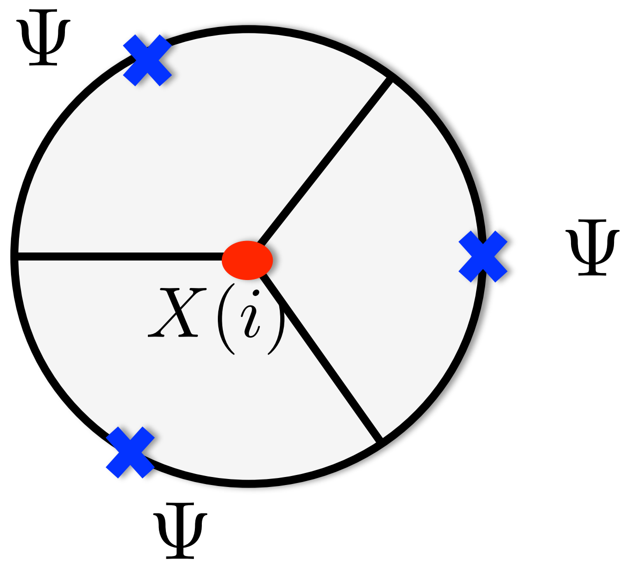
Attempt: Witten's Cubic Action

Witten's Cubic Action :

$$S = \frac{1}{2}\langle \Psi, Q\Psi \rangle + \frac{1}{3}\langle \Psi, X(i)\big(\Psi * \Psi\big)\rangle$$

$X(i)$: Picture Changing Operator (Mid Point !!)

$\langle \Psi, X(i)\big(\Psi * \Psi\big)\rangle$ =



This Product is Associative !!

But contact terms become DIVERGENT !!

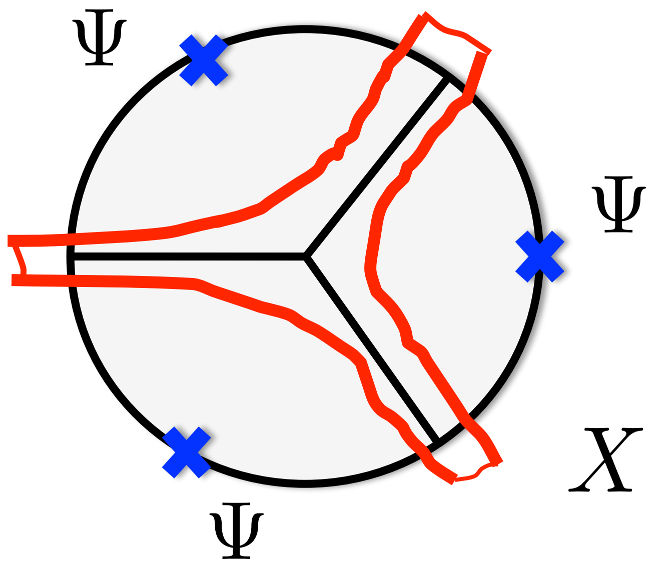
Resolving Witten's Theory (2014)

Using line integral $X = \int \frac{dz}{2\pi i} f(z) X(z)$, we introduce

New product :

$$M_2(\Psi_1, \Psi_2) = \frac{1}{3}\Big(X(\Psi_1 * \Psi_2) + (X\Psi_1) * \Psi_2 + \Psi_1 * (X\Psi_2) \Big)$$

$\langle \Psi, M_2(\Psi^2) \rangle$ =



Non-Associative !!

$$M_2(M_2(A, B), C) \neq M_2(A, M_2(B, C))$$

How to Resolve? (2014)

An A_∞/L_∞-algebra of the (super-)string products.

$$(Q + M_2 + M_3 + \dots)^2 = 0$$

Seeking "Higher" product M_3 satisfying

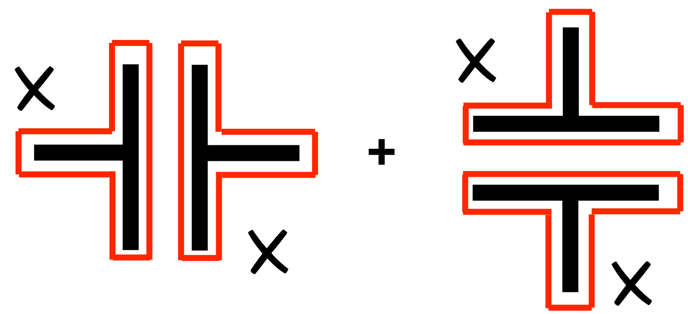
$$M_2^2 + [\![Q, M_3]\!] = 0 \qquad \text{(up to } O(\psi^3) \text{) .}$$

How to Resolve? (2014)

Seeking "Higher" product M_3 satisfying

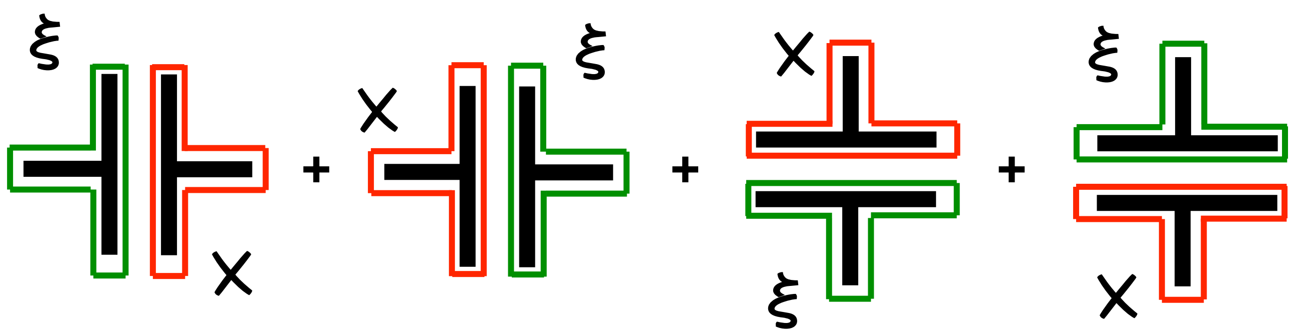
$$(Q + M_2 + M_3 + \dots)^2 = 0 \qquad \text{i.e.} \qquad M_2^2 + [\![Q, M_3]\!] = 0$$

M_2^2 =



Since X = [[Q , ξ]] , we can always construct

M_3 =



Higher Vertices as Regulators (2014)

We can construct "Higher" products $Q + M_2 + M_3 + M_4 + \ldots$

satisfying $(Q + M_2 + M_3 + M_4 + \ldots)^2 = 0$

Note that we used " ξ " but Theory is "small".

We impose $[[\, \eta \,,\ M_2 + M_3 + \cdots \,]] = 0$

→ Vertices $Q + M_2 + M_3 + M_4 + \cdots$ (Not unique)

Infinite Vertices Appear (2014)

Small-space Action for NS Superstring field theory

$$S = \frac{1}{2}\langle \Psi, Q\Psi \rangle + \frac{1}{3}\langle \Psi, M_2(\Psi^2)\rangle + \frac{1}{4}\langle \Psi, M_3(\Psi^3)\rangle + \frac{1}{5}\langle \Psi, M_4(\Psi^4)\rangle + \dots$$

Infinite Vertices $M_2 + M_3 + M_4 + \dots$ appear !!

(Interacting Point is the Mid Point !!)

Gauge Invariance = Nilpotency of Vertices

$$(Q + M_2 + M_3 + M_4 + \dots)^2 = 0$$

2. NS-NS actions

A 'small-space' NS-NS action

Recall the construction of NS Superstring products . . .

$$\mathbf{L}_2^{(0)} \xrightarrow{\xi} \mathbf{L}_2^{(1)}$$

$$\mathbf{L}_3^{(0)} \xrightarrow{\xi} \mathbf{L}_3^{(1)} \xrightarrow{\xi} \mathbf{L}_3^{(2)}$$

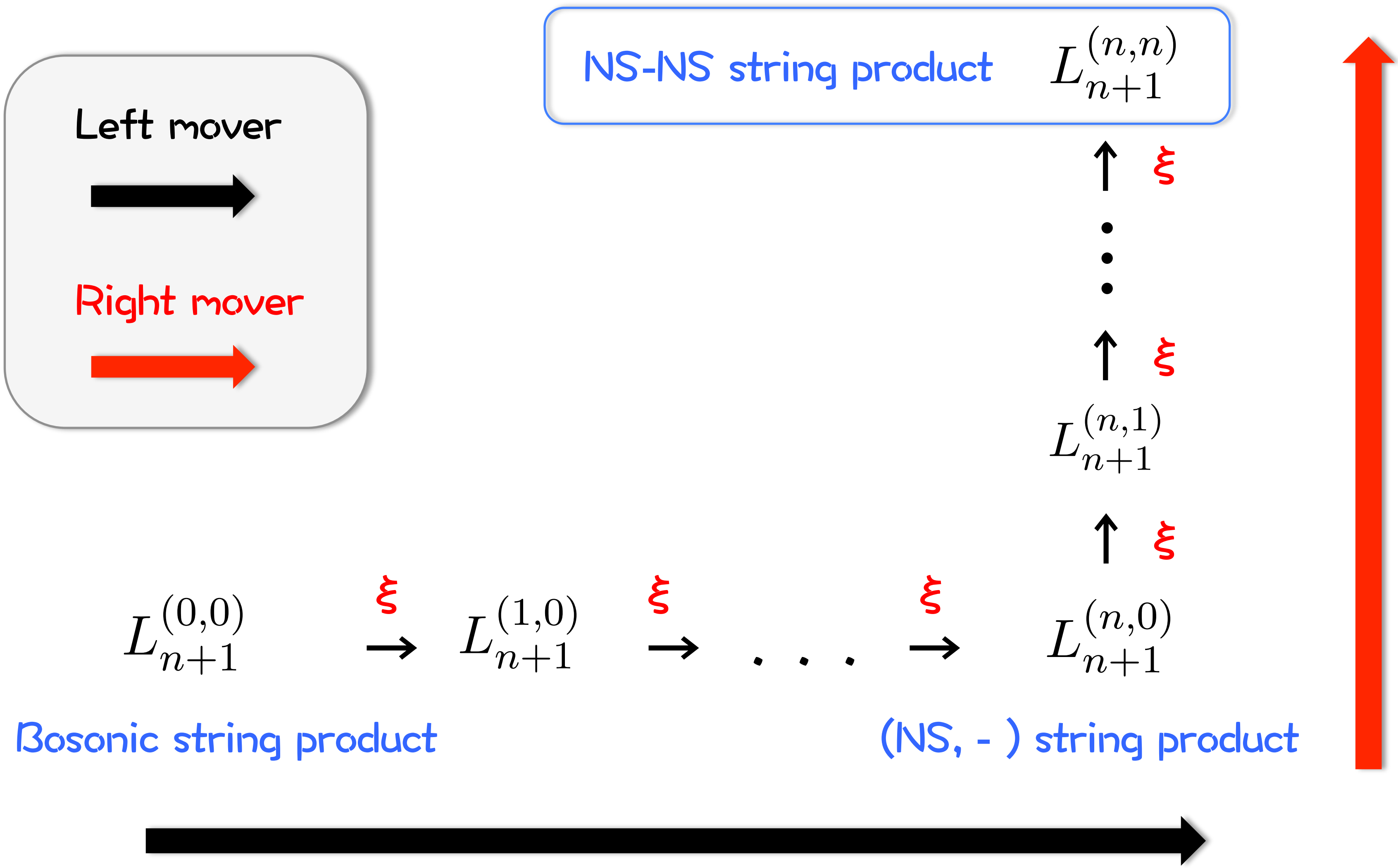
$$\iddots \qquad\qquad \ddots$$

$$\mathbf{L}_{n+1}^{(0)} \xrightarrow{\xi} \mathbf{L}_{n+1}^{(1)} \xrightarrow{\xi} \cdots \xrightarrow{\xi} \mathbf{L}_{n+1}^{(n)}$$

Bosonic string product — NS string product

Repeating this process, NS-NS products appear !!

A 'small-space' NS-NS products



How to obtain WZW-type NS-NS action ?

NS-NS Action

Vertex Op.		String Field	($\#_{gh}$ | $\#_L$, $\#_R$)
$\mathcal{V}(z,\bar{z}) = \xi\bar{\xi}c\bar{c}e^{-\phi}e^{-\bar{\phi}}V_m$	➡	Ψ	$(0\|0,0)$

In the left & right "large" Hilbert space. . .

<A, B> = 0 except for ($\#_{gh}$ | $\#_L$, $\#_R$) of A + B = (3 | -1, -1)

We can construct free action : $S_2 = \frac{1}{2}\langle\bar{\eta}\Psi, Q\eta\Psi\rangle$

→ Can we construct interacting terms ??

→ Pure-gauge . . . ?

Bosonic pure-gauge does not work well

We can construct a 'bosonic' pure-gauge

$$\frac{\partial}{\partial t}\mathcal{G}_{\mathcal{B}}(t) = Q_{\mathcal{G}_B(t)}(\underline{\eta X \Psi})$$

Gauge parameter ?

→ Defining Eq. is NOT unique !?

Attempt : [Using a 'bosonic' pure-gauge]

$$S = \int_0^1 dt \int_0^1 d\kappa \langle \eta\bar{\eta}\Psi, Q_{\mathcal{G}_{\mathcal{B}}(t)}\Psi \rangle$$

→ Nonlinear Gauge Inv. is NOT clear. . .

Add terms to be gauge inv.

Attempt : [Using a 'bosonic' pure-gauge]

$$S = \int_0^1 dt \int_0^1 d\kappa \langle \eta\bar{\eta}\Psi, Q_{\mathcal{G}_{\mathcal{B}}(t)}\Psi \rangle$$

+ appropriate terms

→ Gauge Inv. Action .

For example . . . $O(\psi^3)$

Attempt : [Using a 'bosonic' pure-gauge]

$$S = \frac{1}{2}\langle \bar{\eta}\Psi, Q\eta\Psi\rangle + \frac{\kappa}{3!}\langle \bar{\eta}\Psi, \big[XQ\eta\Psi, \eta\Psi\big]\rangle + O(\kappa^2)$$

+ appropriate terms :

$$\frac{\kappa}{3\cdot 3!}\langle \bar{\eta}\Psi, X\big[Q\eta\Psi, \eta\Psi\big] - 2\big[XQ\eta\Psi, \eta\Psi\big] + \big[Q\eta\Psi, X\eta\Psi\big]\rangle$$

→ Gauge Inv. Action !!

For example . . . $O(\psi^3)$

Gauge invariant action :

$$S = \frac{1}{2}\langle \bar{\eta}\Psi, Q\eta\Psi\rangle + \frac{\kappa}{3!}\langle \bar{\eta}\Psi, \big[Q\eta\Psi, \eta\Psi\big]^L\rangle + O(\kappa^2)$$

where the new 3-product is defined by

$$[A, B]^L := \frac{1}{3}\Big(X[A,B] + [XA, B] + [A, XB]\Big)$$

→ Similarly, we can obtain higher terms . . .

But complicated !!

How to obtain a closed form expression ?

Note that $\eta\,\psi$ appears as a gauge parameter !!

$$S = \frac{1}{2}\langle \bar{\eta}\Psi, Q\underline{\eta\Psi}\rangle + \frac{\kappa}{3!}\langle \bar{\eta}\Psi, \big[Q\underline{\eta\Psi}, \underline{\eta\Psi}\big]^{L}\rangle + O(\kappa^2)$$

- ($\#_{gh}$ | $\#_{pic}{}^{L}$, $\#_{pic}{}^{R}$) of $\eta\,\psi$ = (1 | -1 , 0)

→ Gauge parameter Λ of small-space NS theory

→ We can use a 'small-space' NS pure-gauge !!

A 'small-space' NS pure-gauge

N=1 "small"-space action

$$S = \frac{1}{2}\langle \Phi, Q\Phi \rangle + \sum_{n=1}^{\infty} \frac{\kappa^n}{(n+2)!}\langle \Phi, [\Phi^n, \Phi]^L \rangle$$

which is inv. under the gauge transf.

$$\delta\Phi = Q\Lambda + \sum_{n=1}^{\infty} \frac{\kappa^n}{n!}\left[\Phi^n, \Lambda\right]^L$$

→ Same as bosonic theory. . . (L_∞-relations hold!!)

The pure-gauge is a solution of the differential eq.

$$\frac{\partial}{\partial t}\mathcal{G}(t) = Q\Lambda + \sum_{n=1}^{\infty} \frac{\kappa^n}{n!}\left[\mathcal{G}(t)^n, \Lambda\right]^L$$

$$= Q_{\mathcal{G}(t)}\Lambda$$

NS pure-gauge gives the previous terms!!

Our Answer to Q : How to make N=2 action?

1. Consider a "small"-space NS pure-gauge .

$$\frac{\partial}{\partial t}\mathcal{G}_L(t) = Q_{\mathcal{G}_L(t)}\Lambda \qquad \Lambda \in \mathrm{Ker}\big[\eta\big]$$

2. Identify "large"-space NS-NS string fields ψ

with "small"-space NS gauge parameters Λ .

$$\eta\Psi = \Lambda_{(1|-1,0)}$$

Then, we obtain the NS-NS action.

$$S = \int_0^1 dt \langle \bar{\eta}\Psi, \mathcal{G}_L(t)\rangle \quad \ldots$$ WZW-like !!

Action for "large"-space NS-NS SFT

WZW-like NS-NS Action

$$S = \frac{2}{\alpha'} \int_0^1 dt \langle \bar{\eta}\Psi_t, \mathcal{G}_L(t) \rangle$$

Defining equations

$$\frac{\partial}{\partial \tau}\mathcal{G}_L(\tau\eta\Psi) = Q_{\mathcal{G}_L}(\eta\Psi)$$

$$\frac{\partial}{\partial \tau}\Psi_{\mathbb{X}} = (-1)^{\mathbb{X}}\mathbb{X}\Psi + \kappa[\eta\Psi, \Psi_{\mathbb{X}}]^L$$

Familiar WZW form

$$S|_{\Psi(t)=t\Psi} = \frac{1}{\alpha'}\Big(\langle \Psi_{\bar{\eta}}, \mathcal{G}_L \rangle + \kappa \int_0^1 dt \langle \Psi_t, [\Psi_{\bar{\eta}}(t), \mathcal{G}_L(t)]^L_{\mathcal{G}_L} \rangle \Big)$$

Equation of motion

$$\bar{\eta}\,\mathcal{G}_L = \int_0^1 d\tau \Big(\eta\bar{\eta} Q_{\mathcal{G}_L}\Psi \Big) = 0$$

Nonlinear gauge invariance

Variation of action (t - independent !!)

$$\delta S = \int_0^1 dt \Big(\langle \delta\Psi_t, \bar{\eta}\mathcal{G}_L(t) \rangle + \langle \Psi_t, \delta(\bar{\eta}\mathcal{G}_L(t)) \rangle \Big)$$

$$= \int_0^1 dt\, \partial_t \langle \Psi_\delta(t), \bar{\eta}\mathcal{G}_L(t) \rangle = \langle \Psi_\delta, \bar{\eta}\mathcal{G}_L \rangle$$

$\bar{\eta}\mathcal{G}_L$ is a $Q_{\mathcal{G}_L}$-, η-, and $\bar{\eta}$-exact state

Nonlinear gauge transformation

$$\Psi_\delta = Q_{\mathcal{G}_L}\Lambda + \bar{\eta}\,\Omega \;\; \big(+ \eta\Omega' \big)$$

Field redefinition

$$\widetilde{\Omega} \equiv \Omega' + \tfrac{\kappa}{2}[\eta\Psi, \widetilde{\Omega}]^L + \dots .$$

Linear transf.

$$\delta_\eta \Psi = \eta\, \widetilde{\Omega}$$

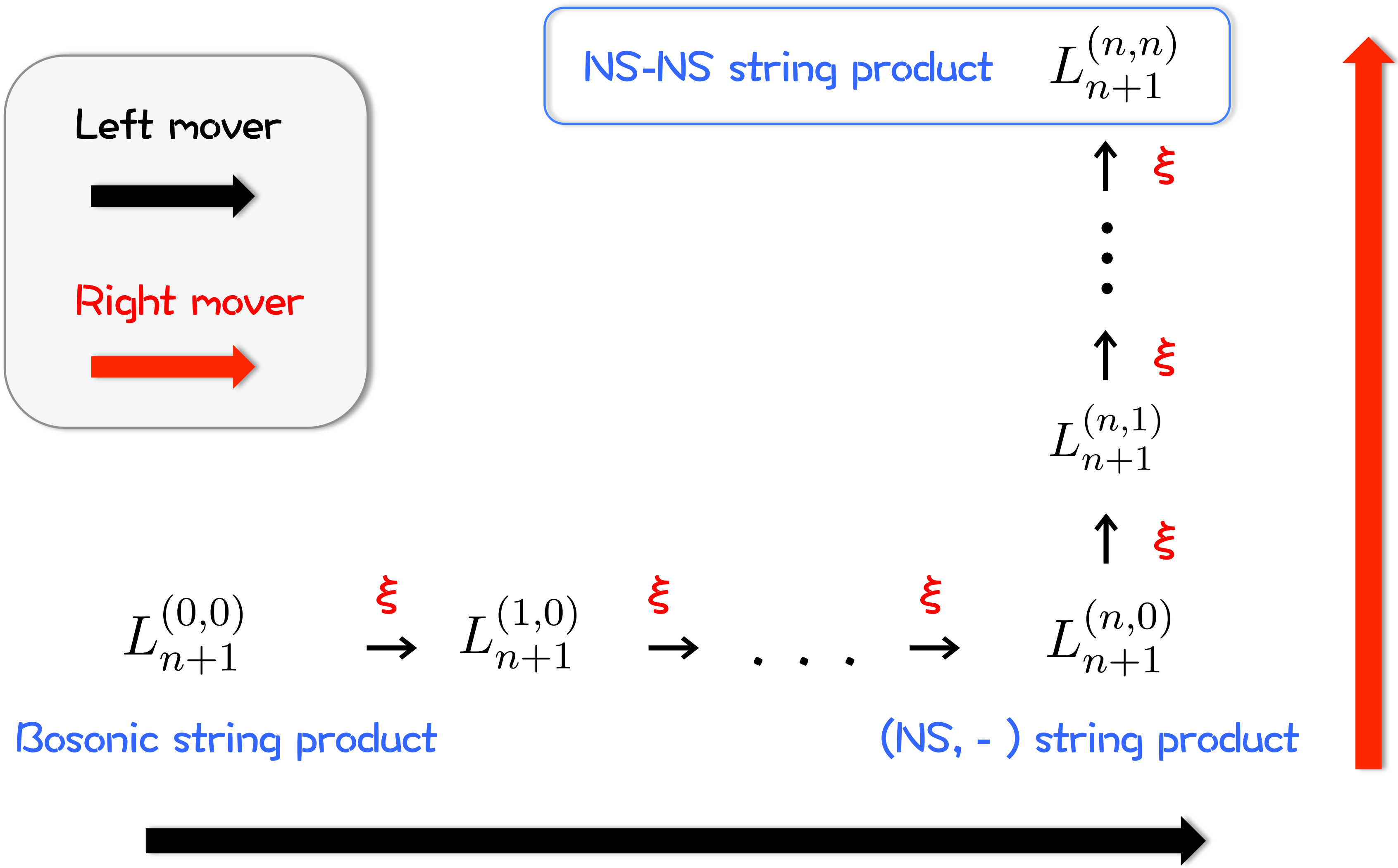
Solving the inverse, we find. . .

$$\delta_\Omega \Psi = \bar{\eta}\Omega + \frac{\kappa}{2}[\eta\Psi, \bar{\eta}\Omega]^L + \frac{\kappa^2}{3}\Big[\bar{\eta}\Omega, Q\eta\Psi, \eta\Psi\Big]^L + \frac{\kappa^2}{12}\Big[[\bar{\eta}\Omega, \eta\Psi], \eta, \Psi\Big]^L + O(\kappa^3)$$

So , . . .

in the NS-NS sector , . . .

A 'large-space' NS-NS action is given by



A 'small-space' NS-NS action is given by

Left mover

Large (Right)

(NS, -) Pure-gauge $\mathcal{G}_L$

(NS, -) gauge parameter

$\eta\,\psi$

$$L_{n+1}^{(0,0)} \overset{\xi}{\rightarrow} L_{n+1}^{(1,0)} \overset{\xi}{\rightarrow} \cdots \overset{\xi}{\rightarrow} L_{n+1}^{(n,0)}$$

Bosonic string product

(NS, -) string product

3. Gauge fixing of WZW-like actions

Strategy

WZW-like actions

↓ Partial Gauge Fixing

A_∞/L_∞-type actions

(Classical BV . . . OK !)

Relation between 'small' and 'large' actions

Large-space Actions for NS & NS-NS

$$S_2 = \frac{1}{2}\langle \eta V, QV \rangle$$

- Q-gauge sym.
- η-gauge sym.

($\bar{\eta}$-gauge sym.)

Partial Gauge Fixing

$$V = \xi \Phi$$

Small-space Actions for NS & NS-NS

$$S_2 = \frac{1}{2}\langle \xi\Phi, Q\Phi \rangle = \frac{1}{2}\langle \Phi, Q\Phi \rangle_{\rm small}$$

- Q-gauge sym.

3-point Interaction

Large-space Actions for NS

$$S = \frac{1}{2}\langle \eta V, QV \rangle + \frac{\kappa}{3!}\langle \eta V, [QV, V] \rangle$$

- Q-gauge sym.
- η-gauge sym.

Partial Gauge Fixing (up to $O(\phi^3)$)

$$V = \xi\Phi + \frac{\kappa}{3!}\xi[\xi\Phi, \Phi] + \mathcal{O}(\kappa^2)$$

Small-space Actions for NS & NS-NS

$$S = \frac{1}{2}\langle \xi\Phi, Q\Psi \rangle + \frac{\kappa}{3!}\langle \xi\Phi, [X\Phi, \Phi] \rangle$$

- Q-gauge sym.

4-point Interaction

Large-space NS Actions

$$S = \int_0^1 dt \langle \eta V, \mathcal{G}(tV) \rangle$$

- Q-gauge sym.
- η-gauge sym.

Partial Gauge Fixing (up to $O(\phi^4)$)

$$V = \xi\Phi + \frac{\kappa}{3!}\xi[\xi\Phi, \Phi]$$
$$+ \frac{\kappa^2}{4!}\Big(\xi\big[\xi\Phi, (Q\xi + X)\Phi, \Phi\big] + \xi\big[\xi[\Phi, \Phi], \xi\Phi\big]$$
$$+ \frac{2}{3}\xi\big[\xi[\xi\Phi, \Phi], \Phi\big] + \frac{2}{3}\big[\xi[\xi\Phi, \Phi], \xi\Phi\big]\Big)$$

Small-space NS Actions

$$S = \frac{1}{2}\langle \Phi, Q\Phi \rangle + \sum_{n=1}^{\infty} \frac{\kappa^n}{(n+2)!} \langle \Phi, [\Phi^n, \Phi]^L \rangle$$

- Q-gauge sym.

For example, in the NS-NS sector . . .

Large-space NS-NS Action

$$S = \frac{2}{\alpha'}\int_0^1 dt\langle\bar{\eta}\Psi_t, \mathcal{G}_L(t)\rangle$$

- Q-gauge sym.
- η-gauge sym.
- $\bar{\eta}$-gauge sym.

1 st. η - Gauge Fixing

$$\Psi = \xi\bar{V} \qquad \bar{V} \in \mathrm{Ker}[\eta]$$

Small-space NS-NS Action

$$S = \frac{1}{2}\langle\Phi, Q\Phi\rangle + \sum_{n=1}^{\infty}\frac{\kappa^n}{(n+2)!}\langle\Phi, [\Phi^n, \Phi]^L\rangle$$

- Q-gauge sym.

For example, in the NS-NS sector . . .

Large-space NS-NS Action

$$S = \frac{2}{\alpha'} \int_0^1 dt \langle \bar{\eta} \Psi_t, \mathcal{G}_L(t) \rangle$$

- Q-gauge sym.
- $\bar{\eta}$-gauge sym.

1st. η - Gauge Fixing : $\Psi = \xi \bar{V}$

2nd. Partial $\bar{\eta}$ - Gauge Fixing (up to $O(\phi^4)$)

$$\bar{V} = \bar{\xi}\Phi + \frac{\kappa}{3!}\bar{\xi}[\bar{\xi}\Phi, \Phi]^L + \frac{\kappa^2}{4!}\Big(\bar{\xi}\big[\bar{\xi}\Phi, (Q\bar{\xi} + X)\Phi, \Phi\big]^L + \bar{\xi}\big[\bar{\xi}[\Phi, \Phi]^L, \bar{\xi}\Phi\big]^L + \frac{2}{3}\bar{\xi}\big[\bar{\xi}[\bar{\xi}\Phi, \Phi]^L, \Phi\big] + \frac{2}{3}\big[\bar{\xi}[\bar{\xi}\Phi, \Phi], \bar{\xi}\Phi\big]^L\Big)$$

Small-space NS-NS Actions

$$S = \frac{1}{2}\langle \Phi, Q\Phi \rangle + \sum_{n=1}^{\infty} \frac{\kappa^n}{(n+2)!} \langle \Phi, [\Phi^n, \Phi]^L \rangle$$

- Q-gauge sym.

Up to $O(\phi^4)$

Witten's Cubic Theory

Witten

Small-space theory (A_∞/L_∞-type actions)		Large-space theory (WZW-type actions)
NS open	⟷	NS open **Berkovits 95'**
NS closed	⟷	NS closed **Berkovits, Okawa, Zwiebach 04'**
NS-NS closed **Erler, Konopka, Sachs 14'**	⟷	NS-NS closed **H.M 14'**

Gauge fixing . . . OK

Gauge fixing . . . OK

How to obtain a closed form expression ?

Construction of a cyclic A_∞/L_∞-isomorphism

- Let $(S(H), L, \omega)$ and $(S(H)', L', \omega')$ be cyclic L_∞-algebras.

L_∞-morphism : A morphism of coalgebra $f : S(H) \rightarrow S(H)'$

satisfying $f L = L' f$.

Cyclic L_∞-morphism : L_∞-morphism f satisfying

$$\omega (A , B) = \omega' (f_1 (A), f_1 (B)) \quad \text{and}$$

$$\Sigma\, \omega' (f_j (A_1, , A_j) , f_k (B_1, , B_k)) = 0 .$$

→ f preserve the Equation of Motion !!

Consider two EOMs

A_∞/L_∞-type EOM

$$(Q + L_2 + L_3 + \ldots)\, e^{\wedge\phi} = 0$$

$$\Updownarrow$$

$$L\, e^{\wedge\phi} = 0$$

- L_∞-algebra -

$$(Q + L_2 + L_3 + \cdots)^2 = 0$$

WZW-type EOM

$$Q_g\, \psi_\eta = 0$$

- Trivial L_∞-algebra -

$$(Q_g + 0 + \cdots)^2 = 0$$

→ Find a isomorphism satisfying $f\, L = Q_g\, f$.

Such a 'f' exists.

$$f = e^{\int dt [v,\ ]_g}\, e^{\int dt \Xi} \qquad \text{(path-ordered exponential)}$$

- Ξ is the gauge products of Erler-Konopka-Suchs .

$$L = (e^{\int dt \Xi})^{\dagger}\, Q\, e^{\int dt \Xi} \qquad e^{\int dt \Xi} (e^{\int dt \Xi})^{\dagger} = 1 = (e^{\int dt \Xi})^{\dagger}\, e^{\int dt \Xi}$$

- $E := e^{\int dt [v,\ ]_g}$ is defined by $\partial_t E(t) = [\, V \,,\, E(t)\,]_{g(t)}$.

$$Q_g = e^{\int dt [v,\ ]_g}\, Q\, (e^{\int dt [v,\ ]_g})^{\dagger} \qquad E\, E^{\dagger} = 1 = E^{\dagger}\, E$$

$$f\, L = EQ\, e^{\int dt \Xi} = (EQE^{\dagger})\, (E\, e^{\int dt \Xi}) = Q_g\, f$$

'f' gives partial gauge fixing conditions.

In the actions, $f = e^{\int dt [v,\ ]_g}\, e^{\int dt\, \Xi}$ gives

$$\int dt\, \left(e^{\int dt [v,\ ]_g}\right)^{\dagger} \eta V = e^{\int dt\, \Xi}\, e^{\wedge \phi}$$

Solving this relation with $V = \xi \eta V$, we obtain

$$V = \xi\Phi + \frac{\kappa}{3!}\xi[\xi\Phi,\Phi] + \frac{\kappa^2}{4!}\Big(\xi\big[\xi\Phi,(Q\xi+X)\Phi,\Phi\big] + \xi\big[\xi[\Phi,\Phi],\xi\Phi\big]$$

$$+ \frac{2}{3}\xi\big[\xi[\xi\Phi,\Phi],\Phi\big] + \frac{2}{3}\big[\xi[\xi\Phi,\Phi],\xi\Phi\big]\Big) + \ldots\ldots$$

Summary

NS and NS-NS sector of Superstring field theories

- Consistent gauge-inv. NS-NS "large"-space action also exists!

  → WZW-like NS-NS action

- Relation between "small"- and "large"- space theories

  Cyclic A_∞/L_∞-isomorphism

  WZW-type ⟷ A_∞/L_∞-type

  → Equivalence of two actions

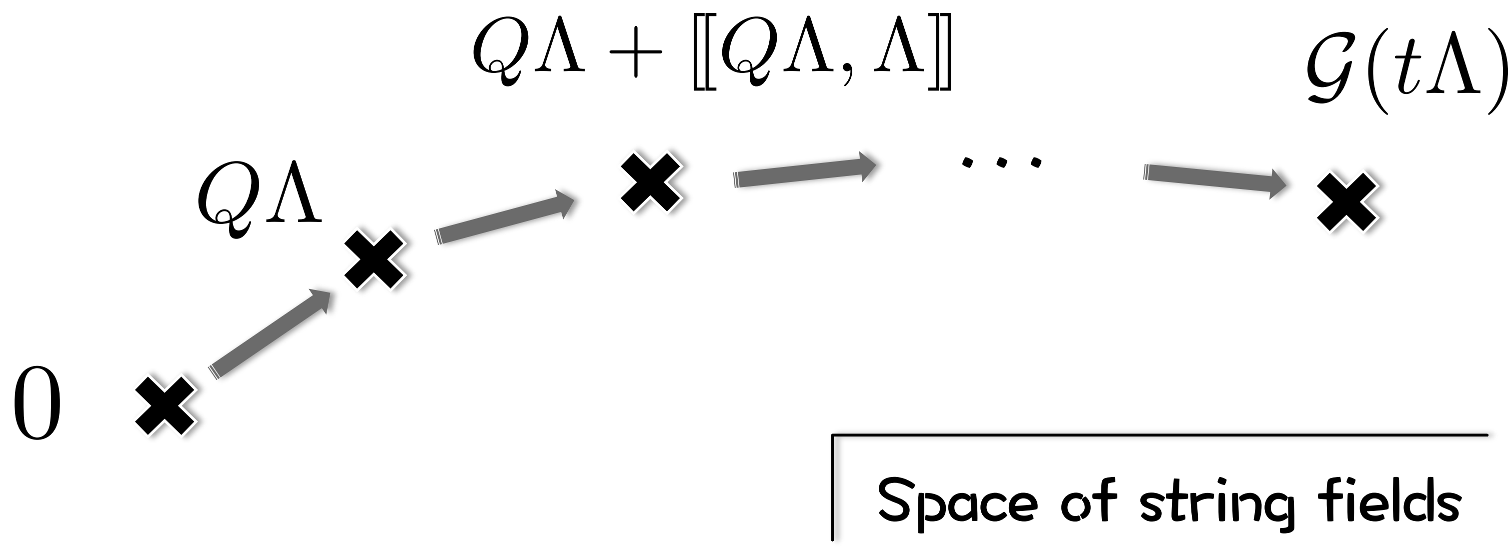
  → Partial gauge fixing conditions appear !!

Differential Eq. gives "Pure-gauge"

Recall the gauge transf. $\delta\Psi = Q_\Psi \Lambda = Q\Lambda + [\![\Psi, \Lambda]\!]$

Shifted BRST op. $Q_\Psi = Q + [\![\Psi, \quad]\!]$ appears!!

Pure-gauge : successive infinitesimal gauge transf. around itself



Differential Eq. gives "Pure-gauge"

Recall the gauge transf. $\delta\Psi = \underline{Q_\Psi \Lambda} = Q\Lambda + [\![\Psi, \Lambda]\!]$

Shifted BRST op. $Q_\Psi = Q + [\![\Psi, \quad]\!]$ appears!!

Pure-gauge : successive infinitesimal gauge transf. around itself

$$\mathcal{G}(t\Lambda + dt\Lambda) - \mathcal{G}(t\Lambda) = Q(dt\Lambda) + [\![\mathcal{G}(t\Lambda), dt\Lambda]\!]$$

$$= Q_{\mathcal{G}(t\Lambda)}(dt\Lambda)$$

The Defining Eq. of Pure-gauge : $\dfrac{\partial}{\partial t}\mathcal{G}(t\Lambda) = Q_{\mathcal{G}(t\Lambda)}\Lambda$

For example . . .

$$\partial_t e^{-t\Lambda} Q e^{t\Lambda} = -\Lambda\big(e^{-t\Lambda} Q e^{t\Lambda}\big) + e^{-t\Lambda} Q\big(e^{t\Lambda} * \Lambda\big)$$

$$= Q\Lambda + [\![e^{-t\Lambda} Q e^{t\Lambda}, \Lambda]\!]$$

Familiar Pure-gauge field $e^{-\Lambda} Q e^{\Lambda}$ is a solution of

the Defining Eq. of Pure-gauge $\frac{\partial}{\partial t}\mathcal{G}(t\Lambda) = Q_{\mathcal{G}(t\Lambda)}\Lambda$!!

In the rest, "Pure-gauge" means "a solution of this Eq." .

Bosonic closed String Field Theory

Zwiebach's Action :

L_∞ - algebra appears !!

$$S = \frac{1}{2}\langle \Psi, Q\Psi \rangle + \frac{\kappa}{3!}\langle \Psi, [\Psi, \Psi] \rangle + \sum_{n=3}^{\infty} \frac{\kappa^n}{(n+1)!}\langle \Psi, [\Psi, \dots, \Psi]_n \rangle$$

Gauge transf. :

$$\delta\Psi = Q\Lambda + \kappa[\Psi, \Lambda] + \frac{\kappa^2}{2}[\Psi, \Psi, \Lambda] + \frac{\kappa^3}{3!}[\Psi, \Psi, \Psi, \Lambda] \dots$$

$$= Q_\Psi \Lambda$$

Generator : Shifted BRST $\quad Q_\Psi \Lambda = Q\Lambda + \sum_{n=1}^{\infty} \frac{\kappa^n}{n!}[\Psi^n, \Lambda]$