# Symmetries and Feynman Rules for Ramond Sector in WZW-type Superstring Field Theories

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## 1. Introduction

### ♠ WZW-type Superstring field theory

◇ A formulation with *no explicit picture changing operator* and working well for NS sector:

- ◦ Open [Berkovits (1995)] / Hetetrotic [Okawa and Zwiebach (2004), Berkovits, Okawa and Zwiebach (2004)]

◇ Difficult to construct an action for R sector

- ◦ EOM [Berkovits (2001)] [HK (2014)]
- • A pseudo-action [Michishita (2005)] [HK (2014)]
  We can *construct* the (self-dual) Feynman rules reproducing the on-shell four point amplitudes.

However, these rules do not reproduce the five-point amplitudes [Michishita (2007)], or have an ambiguity [HK (2013)].

**Q: Can we construct consistent Feynman rules?**

# Plan of the Talk

1. Introduction
2. New Feynman rules in the WZW-type open SSFT
   2.1 EOM and the pseudo-action
   2.2 Gauge fixing and the self-dual Feynman rules
   2.3 Gauge symmetries and the new Feynman rules
3. Amplitudes with the external fermions
4. Summary and discussion

# 2. New Feynman rules in the WZW-type Open SSFT

## 2.1 EOM [Berkovits] and the pseudo-action [Michishita]

String fields : Grassmann even, NS $\Phi \in \mathcal{H}_{large}^{(NS)}$ and R $\Psi \in \mathcal{H}_{large}^{(R)}$ with $(G,P) = (0,0)$ and $(0,1/2)$, respectively.

NS action :

$$S_{NS} = \frac{1}{2}\langle (e^{-\Phi}(Qe^{\Phi}))(e^{-\Phi}(\eta e^{\Phi})) - \int_0^1 dt (e^{-t\Phi}(\partial_t e^{t\Phi}))\{(e^{-t\Phi}(Qe^{t\Phi})), (e^{-t\Phi}(\eta e^{t\Phi}))\}\rangle. \tag{1}$$

EOM including the R sector :

$$\eta(e^{-\Phi}(Qe^{\Phi})) + (\eta\Psi)^2 = 0, \qquad Q'\eta\Psi = 0, \tag{2}$$

where $Q'A = QA + A(e^{-\Phi}(Qe^{\Phi})) - (-1)^{|A|}(e^{-\Phi}(Qe^{\Phi}))A$.

Gauge symmetry :

$$e^{-\Phi}(\delta e^{\Phi}) = Q'\Lambda_0 + \eta\Lambda_1 - (\eta\Psi)\Lambda_{\frac{1}{2}} - \Lambda_{\frac{1}{2}}(\eta\Psi), \tag{3a}$$

$$\delta\Psi = Q'\Lambda_{\frac{1}{2}} + \eta\Lambda_{\frac{3}{2}} + \Psi(\eta\Lambda_1) - (\eta\Lambda_1)\Psi. \tag{3b}$$

Auxiliary field : Grassmann even, $\Xi \in \mathcal{H}^{(R)}_{large}$ with $(G, P) = (0, -1/2)$.

The pseudo-action :

$$S_R = -\frac{1}{2}\langle (Q\Xi)e^{\Phi}(\eta\Psi)e^{-\Phi}\rangle. \qquad (4)$$

The variation of $S = S_{NS} + S_R$ yields

$$\eta(e^{-\Phi}(Qe^{\Phi})) + \frac{1}{2}\{\eta\Psi, Q'\Xi'\} = 0, \qquad Q'\eta\Psi = 0, \qquad \eta Q'\Xi' = 0. \qquad (5)$$

where $\Xi' = e^{-\Phi}\Xi e^{\Phi}$. If we eliminate $\Xi$ by imposing a constraint $Q'\Xi' = \eta\Psi$, (5) become EOM (2).

∘ Since this is not the true action we cannot *derive* the Feynman rules.

## 2.2 Gauge fixing and the self-dual Feynman rules

Quadratic part of the action

$$S = \frac{1}{2}\langle (Q\Phi)(\eta\Phi)\rangle - \frac{1}{2}\langle (Q\Xi)(\eta\Psi)\rangle, \tag{6}$$

has gauge symmetries

$$\delta\Phi = Q\Lambda_0 + \eta\Lambda_1, \qquad \delta\Psi = Q\Lambda_{\frac{1}{2}} + \eta\Lambda_{\frac{3}{2}}, \qquad \delta\Xi = Q\Lambda_{-\frac{1}{2}} + \eta\tilde{\Lambda}_{\frac{1}{2}}. \tag{7}$$

Fix them by (the simplest) gauge conditions

$$b_0\Phi = \xi_0\Phi = 0, \qquad b_0\Psi = \xi_0\Psi = 0, \qquad b_0\Xi = \xi_0\Xi = 0. \tag{8}$$

The propagators become

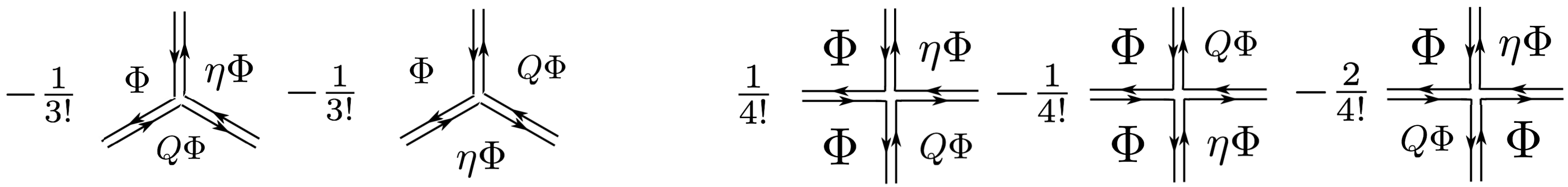
$$\langle\Phi\Phi\rangle \equiv \Pi = \frac{\xi_0 b_0}{L_0} = \int_0^\infty d\tau (\xi_0 b_0) e^{-\tau L_0}, \qquad \langle\Psi\Xi\rangle = \langle\Xi\Psi\rangle = -2\frac{\xi_0 b_0}{L_0} = -2\Pi. \tag{9}$$

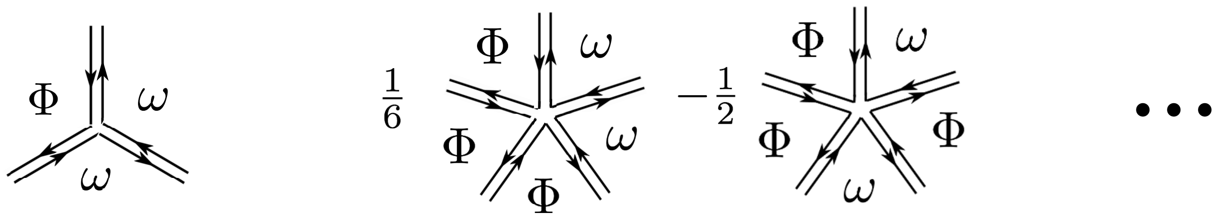
In order to take into account the constraint, we replace $\eta\Psi$ and $Q\Xi$ by their (linearized) "self-dual" part $\omega = (\eta\Psi + Q\Xi)/2$ in vertices: (the self-dual rules).

## Propagator :

$$\text{NS} \;:\; \overline{\Phi\Phi} = \frac{\xi_0 b_0}{L_0}, \qquad \text{R} \;:\; \overline{\omega\omega} = \frac{1}{2}\left(Q\left(\frac{\xi_0 b_0}{L_0}\right)\eta + \eta\left(\frac{\xi_0 b_0}{L_0}\right)Q\right) \quad \text{(diagonal)}$$

## Vertices :




## (on-shell) external fermions : $\omega$

Restrict them by the linearized constraint

$$Q\Xi = \eta\Psi, \qquad (\text{Then} \quad \omega = \eta\Psi) \tag{10}$$

(which automatically implies the on-shell condition $Q\eta\Psi = 0$.)

The self-dual Feynman rules give the well-known (on-shell) 4-point amplitudes but *cannot* reproduce the 5-point amplitudes. [Michishita]

## 2.3 Gauge symmetries and the new Feynman rules

Gauge symmetries :

The (pseudo-) action $S = S_{NS} + S_R$ is invariant under

$$e^{-\Phi}(\delta e^{\Phi}) = Q'\Lambda'_0 + \eta\Lambda_1, \tag{11a}$$

$$\delta\Psi = \eta\Lambda_{\frac{3}{2}} + [\Psi, \eta\Lambda_1], \qquad \delta\Xi = Q\Lambda_{-\frac{1}{2}} + [Q, \Lambda_0]. \tag{11b}$$

These are compatible with the self-dual/anti-self-dual decomposition.

$$\delta(Q'\Xi' \pm \eta\Psi) = [(Q'\Xi' \pm \eta\Psi), \Lambda_1], \tag{12}$$

and so respected by the self-dual rules. However, these symmetries are not enough to gauge away all the un-physical states. They do not contain all the symmetries of the quadratic action (7).

The missing "symmetries" generated by $\Lambda_{\frac{1}{2}}$ and $\tilde{\Lambda}_{\frac{1}{2}}$ can be extended to

$$e^{-\Phi}(\delta e^{\Phi}) = -\frac{1}{2}\{Q'\Xi', \Lambda_{\frac{1}{2}}\} + \frac{1}{2}\{\eta\Psi, \tilde{\Lambda}_{\frac{1}{2}}\}, \tag{13}$$

$$\delta\Psi = Q'\Lambda_{\frac{1}{2}}, \qquad \delta\Xi = e^{\Phi}(\eta\tilde{\Lambda}_{\frac{1}{2}})e^{-\Phi}. \tag{14}$$

The variation of the action under these transformations becomes

$$\delta S = \frac{1}{4}\langle\Lambda_{\frac{1}{2}}\,[(Q'\Xi')^2, (Q'\Xi' - \eta\Psi)]\rangle + \frac{1}{4}\langle\tilde{\Lambda}_{\frac{1}{2}}\,[(\eta\Psi)^2, (Q'\Xi' - \eta\Psi)]\rangle, \tag{15}$$

which vanishes under the constraint.

These additional "symmetries" are not compatible with the self-dual/andti-self-dual decomposition. The self-dual rules do not respect them.

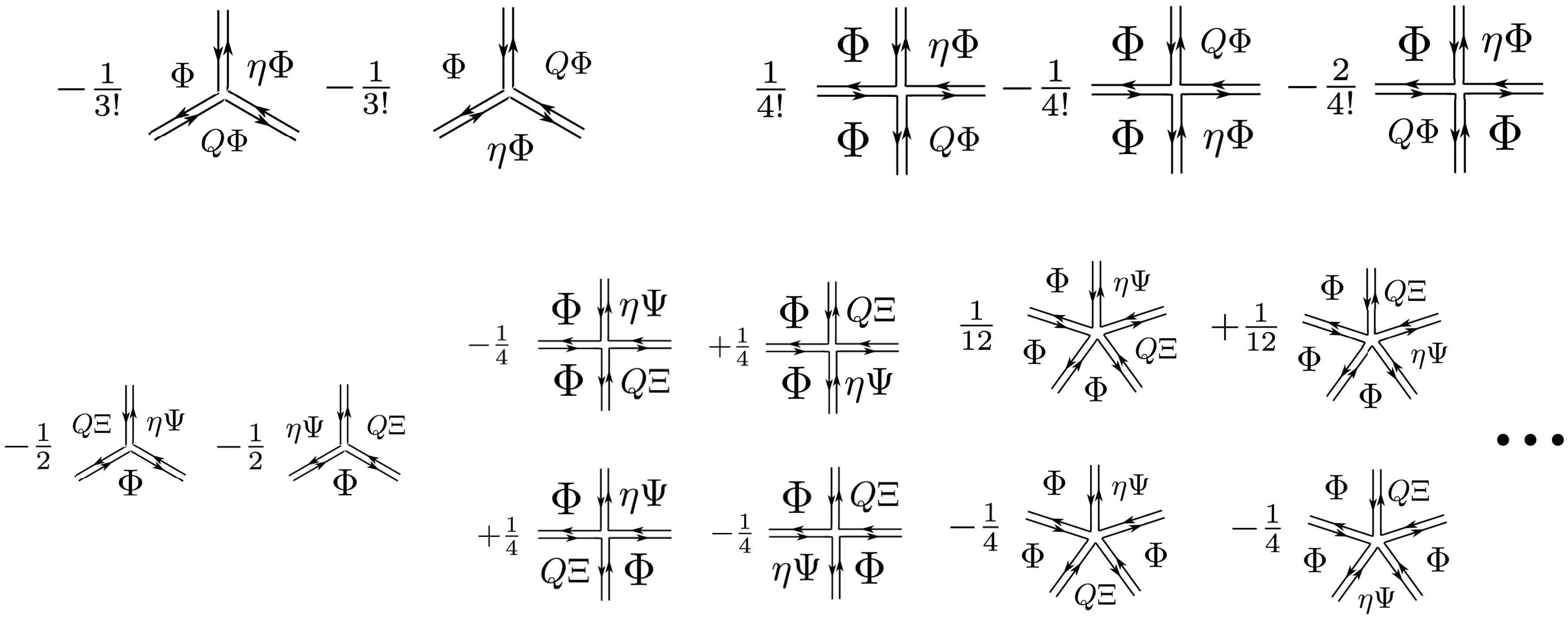
Claim : The Feynman rules have to respect these "symmetries" too.

## The new Feynman rules

### Propagator :

$$\text{NS} \;:\; \overline{\Phi\,\Phi} = \frac{\xi_0 b_0}{L_0}, \qquad \text{R} \;:\; \overline{\Psi\,\Xi} = \overline{\Xi\,\Psi} = -2\frac{\xi_0 b_0}{L_0} \quad \text{(off-diagonal)}$$

### Vertices :



(on-shell) external fermions : Add two possibilities, $\Psi$ and $\Xi$.

Restrict them by the linearized constraint

$$Q\Xi = \eta\Psi. \tag{16}$$

We can show that all the on-shell 4- and 5 point amplitudes are correctly reproduced by the new Feynman rules.

## 3. Amplitudes with the external fermions

Four-point amplitudes are correctly reproduced as in the self-dual rules.

Difference between two rules comes from from the diagram with either

(i) at least two fermion propagators, $\because$

$$\mathcal{A} \sim \langle \cdots (Q\Pi_R\eta + \eta\Pi_R Q) \cdots (Q\Pi_R\eta + \eta\Pi_R Q) \cdots \rangle_W , \tag{17}$$

in the self-dual rules and

$$\mathcal{A} \sim \langle \cdots Q\Pi_R\eta \cdots Q\Pi_R\eta \cdots \rangle_W + \langle \cdots \eta\Pi_R Q \cdots \eta\Pi_R Q \cdots \rangle_W , \tag{18}$$

in the new rules. Or,

(ii) two-fermion-even-boson interaction at least one of whose fermions, $\because$

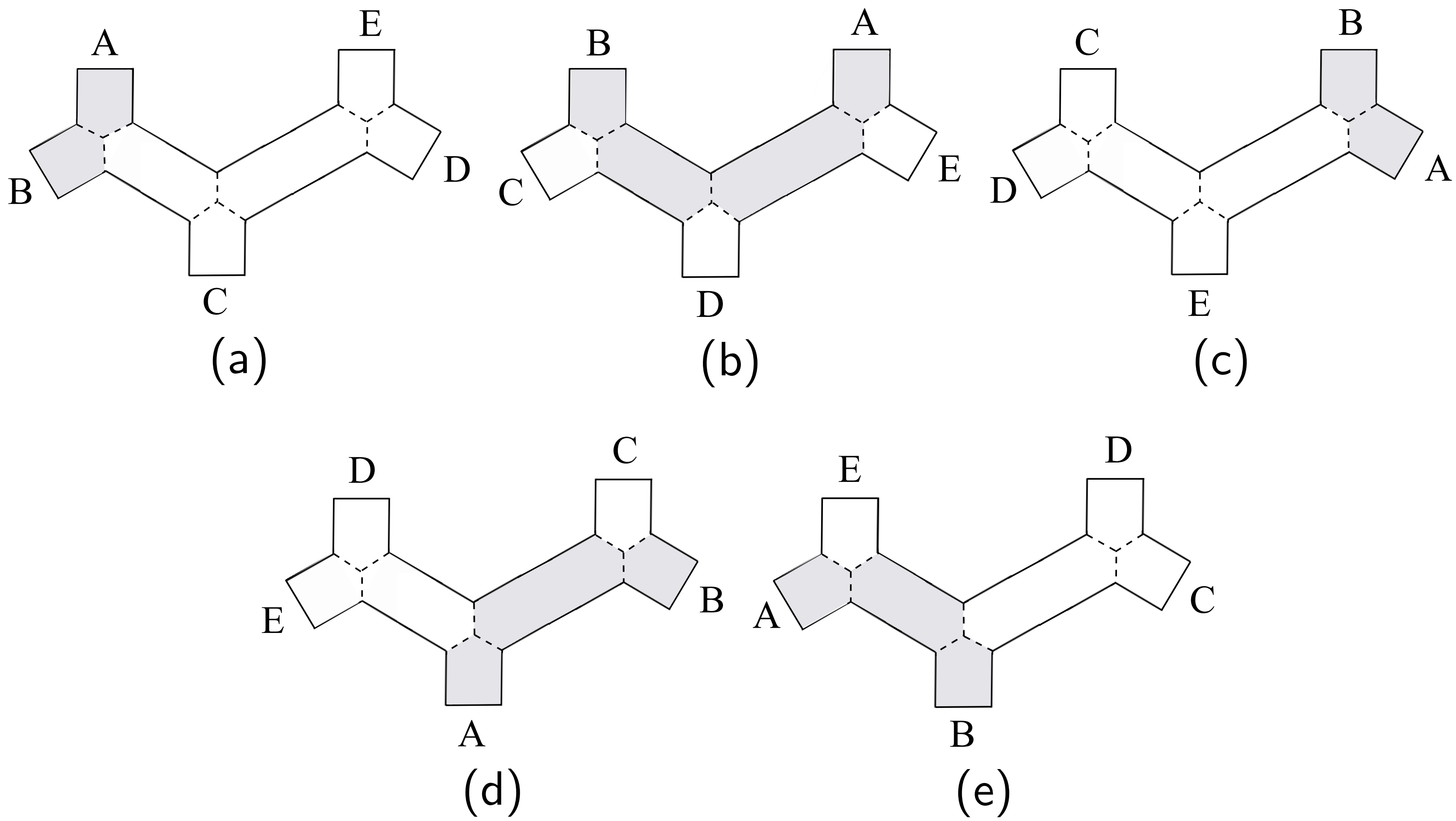
$$e.g. \qquad S_R^{(4)} = -\frac{1}{4}\Big(\langle \Phi^2 (Q\Xi)(\eta\Psi)\rangle - \langle \Phi^2 (\eta\Psi)(Q\Xi)\rangle\Big) + \frac{1}{4}\Big(\langle \Phi(Q\Xi)\Phi(\eta\Psi)\rangle - \langle \Phi(\eta\Psi)\Phi(Q\Xi)\rangle\Big). \tag{19}$$

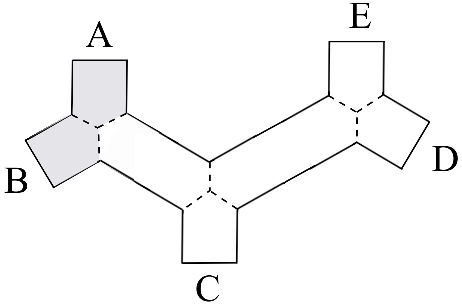
These differences improve the discrepancy in the five-point amplitudes!

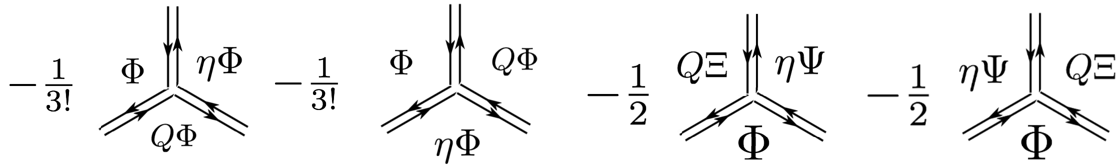
## 3.2 Five-point amplitudes

For example, ordering FFBBB

Five two-propagator (2P) diagrams

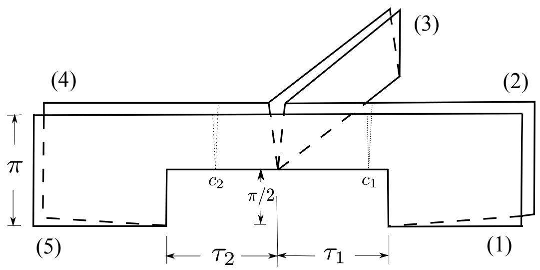





(a)

$$\mathcal{A}^{(2\mathrm{A})(a)}_{FFBBB} = \frac{1}{8}\int_0^\infty d\tau_1 \int_0^\infty d\tau_2$$
$$\times \Bigg( \langle (Q\Xi_A \eta\Psi_B + \eta\Psi_A Q\Xi_B)(\xi_{c_1} b_{c_1} Q)\Phi_C (\eta\xi_{c_2} b_{c_2})(Q\Phi_D \eta\Phi_E + \eta\Phi_D Q\Phi_E)\rangle_W$$
$$+ \langle (Q\Xi_A \eta\Psi_B + \eta\Psi_A Q\Xi_B)(\xi_{c_1} b_{c_1} \eta)\Phi_C (Q\xi_{c_2} b_{c_2})(Q\Phi_D \eta\Phi_E + \eta\Phi_D Q\Phi_E)\rangle_W \Bigg) \quad (20)$$



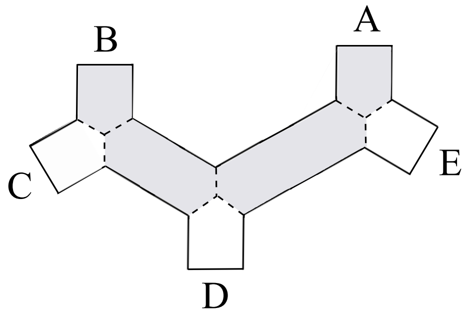

Witten diagram

Using

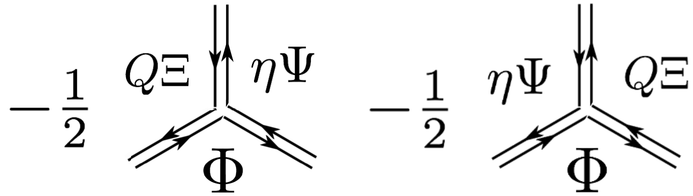
$$\int_0^\infty d\tau \{Q, b_0\} e^{-\tau L_0} = -\int_0^\infty d\tau \frac{\partial}{\partial\tau} e^{-\tau L_0}, \tag{21}$$

One can rewrite it as

$$\begin{aligned}
\mathcal{A}^{(2\mathrm{A})(a)}_{FFBBB} = \frac{1}{2}\int_0^\infty d^2\tau \, \langle \Big(Q\Xi_A \, \eta\Psi_B + \eta\Psi_A \, Q\Xi_B\Big) \, (\xi_{c_1} b_{c_1}) \, Q\Phi_C \, b_{c_2} \, Q\Phi_D \, \eta\Phi_E \rangle_W \\
+ \frac{1}{8}\int_0^\infty d\tau \, \Bigg( \langle \Big(Q\Xi_A \, \eta\Psi_B + \eta\Psi_A \, Q\Xi_B\Big) \\
\times (\xi_c b_c) \, \Phi_C \, \Big(Q\Phi_D \, \eta\Phi_E + \eta\Phi_D \, Q\Phi_E\Big) \rangle_W \\
- 2\langle \Big(Q\Xi_A \, \eta\Psi_B + \eta\Psi_A \, Q\Xi_B\Big) \, (\xi_c b_c) \, Q\Phi_C \, \eta(\Phi_D \, \Phi_E) \rangle_W \\
- \langle \Big(Q\Phi_D \, \eta\Phi_E + \eta\Phi_D \, Q\Phi_E\Big) \\
\times (\xi_c b_c) \, \Big(Q\Xi_A \, \eta\Psi_B + \eta\Psi_A \, Q\Xi_B\Big) \, \Phi_C \rangle_W \\
- 2\langle \eta(\Phi_D \, \Phi_E) \, (\xi_c b_c) \, \Big(Q\Xi_A \, \eta\Psi_B + \eta\Psi_A \, Q\Xi_B\Big) \, Q\Phi_C \rangle_W \Bigg),
\end{aligned} \tag{22}$$




(b)




$$\mathcal{A}^{(2\mathrm{P})(b)}_{FFBBB} = -\frac{1}{2}\int_0^\infty d\tau_1 \int_0^\infty d\tau_2$$
$$\times \Bigg( \langle Q\Xi_B\ \Phi_C\ (\eta\xi_{c_1}b_{c_1}Q)\ \Phi_D\ (\eta\xi_{c_2}b_{c_2}Q)\ \Phi_E\ \eta\Psi_A\rangle_W$$
$$+ \langle \eta\Psi_B\ \Phi_C\ (Q\xi_{c_1}b_{c_1}\eta)\ \Phi_D\ (Q\xi_{c_2}b_{c_2}\eta)\ \Phi_E\ Q\Xi_A\rangle_W \Bigg) \tag{23}$$

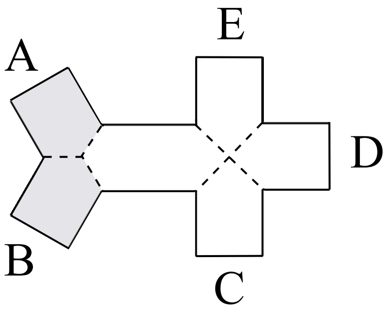
$$
\begin{aligned}
= \frac{1}{2} \int d^2\tau \, \Big( & \langle Q\Xi_B \; Q\Phi_C \; (\xi_{c_1} b_{c_1}) \; Q\Phi_D \; b_{c_2} \; \eta\Phi_E \; \eta\Psi_A \rangle_W \\
& + \langle \eta\Psi_B \; Q\Phi_C \; (\xi_{c_1} b_{c_1}) \; Q\Phi_D \; b_{c_2} \; \eta\Phi_E \; Q\Xi_A \rangle_W \Big) \\
- \frac{1}{2} \int d\tau \Big( & \langle Q\Xi_B \; \Phi_C \; (\xi_c b_c) \; \eta\Phi_D \; Q\Phi_E \; \eta\Psi_A \rangle_W \\
& - \langle Q\Xi_B \; Q\Phi_C \; (\xi_c b_c) \; \eta(\Phi_D \; \Phi_E) \; \eta\Psi_A \rangle_W \\
& - \langle Q\Phi_E \; \eta\Psi_A \; (\xi_c b_c) \; Q\Xi_B \; \eta(\Phi_C \; \Phi_D) \rangle_W \\
& + \langle \eta\Phi_E \; Q\Xi_A \; (\xi_c b_c) \; \eta\Psi_B \; Q\Phi_C \; \Phi_D \rangle_W \\
& + \langle \eta\Phi_E \; \eta\Psi_A \; (\xi_c b_c) \; Q\Xi_B \; Q\Phi_C \; \Phi_D \rangle_W \\
& - \langle \Phi_E \; \eta\Psi_A \; (\xi_c b_c) \; Q\Xi_B \; Q\Phi_C \; \eta\Phi_D \rangle_W \Big) .
\end{aligned} \tag{24}
$$

$$
\begin{aligned}
\mathcal{A}^{(2\mathrm{P})(c)}_{FFBBB} =& \frac{1}{2}\int_0^\infty d^2\tau \ \langle Q\Phi_C \ Q\Phi_D \ (\xi_{c_1} b_{c_1}) \ \eta\Phi_E \ b_{c_2} \ \Big(Q\Xi_A \ \eta\Psi_B + \eta\Psi_A \ Q\Xi_B\Big)\rangle_W \\
&+ \frac{1}{8}\int_0^\infty d\tau \Bigg( 2\langle \Phi_C \ Q\Phi_D \ (\xi_c b_c) \ \eta\Phi_E \ \Big(Q\Xi_A \ \eta\Psi_B + \eta\Psi_A \ Q\Xi_B\Big)\rangle_W \\
&\quad + \langle \Big(Q\Phi_C \ \eta\Phi_D - \eta\Phi_C \ Q\Phi_D\Big) \\
&\qquad\qquad \times (\xi_c b_c) \ \Phi_E \ \Big(Q\Xi_A \ \eta\Psi_B + \eta\Psi_A \ Q\Xi_B\Big)\rangle_W \\
&\quad - 2\langle Q\Phi_C \ \eta\Phi_D \ (\xi_c b_c) \ \Phi_E \ \Big(Q\Xi_A \ \eta\Psi_B + \eta\Psi_A \ Q\Xi_B\Big)\rangle_W \\
&\quad - 2\langle Q\Phi_C \ \Phi_D \ (\xi_c b_c) \ \eta\Phi_E \ \Big(Q\Xi_A \ \eta\Psi_B + \eta\Psi_A \ Q\Xi_B\Big)\rangle_W \\
&\quad - 2\langle \Big(Q\Xi_A \ \eta\Psi_B + \eta\Psi_A \ Q\Xi_B\Big) \ (\xi_c b_c) \ \Phi_C \ Q\Phi_D \ \eta\Phi_E\rangle_W \\
&\quad - \langle \Big(Q\Xi_A \ \eta\Psi_B + \eta\Psi_A \ Q\Xi_B\Big) \\
&\qquad\qquad \times (\xi_c b_c) \ \Big(Q\Phi_C \ \eta\Phi_D - \eta\Phi_C \ Q\Phi_D\Big) \ \Phi_E\rangle_W \\
&\quad + 2\langle \Big(Q\Xi_A \ \eta\Psi_B + \eta\Psi_A \ Q\Xi_B\Big) \ (\xi_c b_c) \ Q\Phi_C \ \eta(\Phi_D \ \Phi_E)\rangle_W \Bigg), \qquad (25)
\end{aligned}
$$

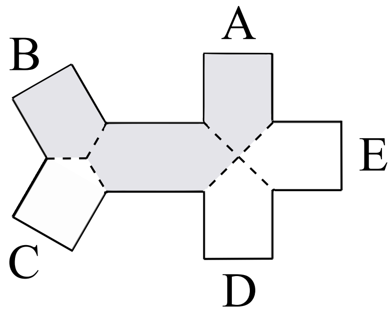
$$
\begin{aligned}
\mathcal{A}^{(2\mathrm{P})(d)}_{FFBBB} =& \frac{1}{2}\int_0^\infty d^2\tau \; \langle Q\Phi_D \; \eta\Phi_E \; (\xi_{c_1} b_{c_1}) \; \Big( Q\Xi_A \; b_{c_2} \; \eta\Psi_B + \eta\Psi_A \; b_{c_2} \; Q\Xi_B \Big) \; Q\Phi_C \rangle_W \\
&+ \frac{1}{4}\int_0^\infty d\tau \Bigg( \langle \Big( Q\Phi_D \; \eta\Phi_E + \eta\Phi_D \; Q\Phi_E \Big) \; (\xi_c b_c) \; \eta\Psi_A \; Q\Xi_B \; \Phi_C \rangle_W \\
&\qquad + \langle \eta(\Phi_D \; \Phi_E) \; (\xi_c b_c) \; \Big( Q\Xi_A \; \eta\Psi_B + \eta\Psi_A \; Q\Xi_B \Big) \; Q\Phi_C \rangle_W \\
&\qquad + \langle Q\Xi_B \; \Phi_C \; (\xi_c b_c) \; \Big( Q\Phi_D \; \eta\Phi_E + \eta\Phi_D \; Q\Phi_E \Big) \; \eta\Psi_A \rangle_W \\
&\qquad - \langle Q\Xi_B \; Q\Phi_C \; (\xi_c b_c) \; \eta(\Phi_D \; \Phi_E) \; \eta\Psi_A \rangle_W \\
&\qquad - \langle \eta\Psi_B \; Q\Phi_C \; (\xi_c b_c) \; \eta(\Phi_D \; \Phi_E) \; Q\Xi_A \rangle_W \Bigg),
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
\mathcal{A}^{(2\mathrm{P})(e)}_{FFBBB} =& \frac{1}{2}\int_0^\infty d^2\tau \ \Big( \langle \eta\Phi_E \ Q\Xi_A \ (\xi_{c_1} b_{c_1}) \ \eta\Psi_B \ b_{c_2} \ Q\Phi_C \ Q\Phi_D \rangle_W \\
& \qquad + \langle \eta\Phi_E \ \eta\Psi_A \ (\xi_{c_1} b_{c_1}) \ Q\Xi_B \ b_{c_2} \ Q\Phi_C \ Q\Phi_D \rangle_W \Big) \\
& - \frac{1}{4}\int_0^\infty d\tau \Big( \langle \Phi_E \ \eta\Psi_A \ (\xi_c b_c) \ Q\Xi_B \ \big( Q\Phi_C \ \eta\Phi_D + \eta\Phi_C \ Q\Phi_D \big) \rangle_W \\
& \qquad - \langle \eta\Phi_E \ Q\Xi_A \ (\xi_c b_c) \ \eta\Psi_B \ \big( Q\Phi_C \ \Phi_D - \Phi_C \ Q\Phi_D \big) \rangle_W \\
& \qquad - \langle \eta\Phi_E \ \eta\Psi_A \ (\xi_c b_c) \ Q\Xi_B \ \big( Q\Phi_C \ \Phi_D - \Phi_C \ Q\Phi_D \big) \rangle_W \\
& \qquad - \langle \big( Q\Phi_C \ \eta\Phi_D + \eta\Phi_C \ Q\Phi_D \big) \ (\xi_c b_c) \ \Phi_E \ \eta\Psi_A \ Q\Xi_B \rangle_W \\
& \qquad - \langle \big( Q\Phi_C \ \Phi_D - \Phi_C \ Q\Phi_D \big) \\
& \qquad \qquad \times (\xi_c b_c) \ \eta\Phi_E \ (Q\Xi_A \ \eta\Psi_B + \eta\Psi_A \ Q\Xi_B) \rangle_W \Big),
\end{aligned}
\tag{27}
$$

five one-propagator diagrams




(a)

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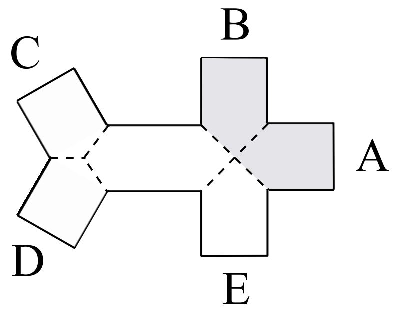


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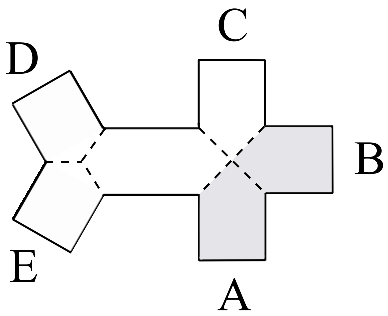


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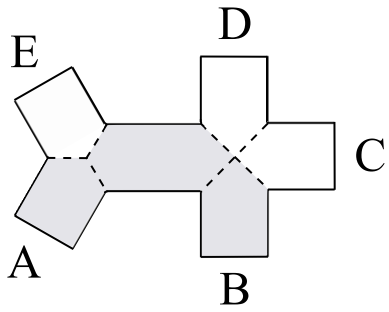



(e)

Similarly,

$$
\begin{aligned}
\mathcal{A}^{(1\mathrm{P})(a)}_{FFBBB} =& \frac{1}{24}\int_0^\infty d\tau \Bigg( \langle \Big( Q\Xi_A\ \eta\Psi_B + \eta\Psi_A\ Q\Xi_B \Big) \\
& \qquad \times\ (\xi_c b_c)\ \Phi_C\ \Big( Q\Phi_D\ \eta\Phi_E - \eta\Phi_D\ Q\Phi_E \Big)\rangle_W \\
& - 2\langle \Big( Q\Xi_A\ \eta\Psi_B + \eta\Psi_A\ Q\Xi_B \Big) \\
& \qquad \times\ (\xi_c b_c)\ \Big( Q\Phi_C\ \Phi_D\ \eta\Phi_E - \eta\Phi_C\ \Phi_D\ Q\Phi_E \Big)\rangle_W \\
& + \langle \Big( Q\Xi_A\ \eta\Psi_B + \eta\Psi_A\ Q\Xi_B \Big) \\
& \qquad \times\ (\xi_c b_c)\ \Big( Q\Phi_C\ \eta\Phi_D - \eta\Phi_C\ Q\Phi_D \Big)\ \Phi_E\rangle_W \Bigg), \qquad (28)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}^{(1\mathrm{P})(b)}_{FFBBB} =& \frac{1}{4}\int_0^\infty d\tau \,\Bigg( \langle \eta\Psi_B\ \eta\Phi_C\ (\xi_c b_c)\ Q(\Phi_D\ \Phi_E)\ Q\Xi_A\rangle_W \\
&\qquad - \langle Q\Xi_B\ \eta\Phi_C\ (\xi_c b_c)\ Q(\Phi_D\ \Phi_E)\ \eta\Psi_A\rangle_W \\
&\qquad - \langle \eta\Psi_B\ \Phi_C\ (\xi_c b_c)\ \Big(Q\Phi_D\ \eta\Phi_E - \eta\Phi_D\ Q\Phi_E\Big)\ Q\Xi_A\rangle_W \Bigg) \\
&- \frac{1}{4}\langle Q\Xi_A\ \eta\Psi_B\ \Phi_C\ \Phi_D\ \Phi_E\rangle_W, \qquad (29)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}^{(1\mathrm{P})(c)}_{FFBBB} =& \frac{1}{8}\int_0^\infty d\tau\, \langle \Big(Q\Phi_C\ \eta\Phi_D + \eta\Phi_C\ Q\Phi_D\Big) \\
&\qquad \times (\xi_c b_c)\ \Phi_E\ \Big(Q\Xi_A\ \eta\Psi_B - \eta\Psi_A\ Q\Xi_B\Big)\rangle_W, \qquad (30)
\end{aligned}
$$

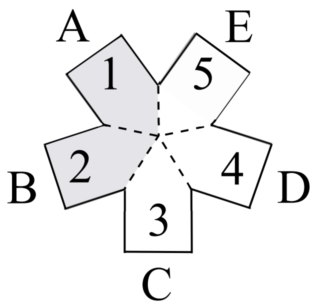
$$
\begin{aligned}
\mathcal{A}^{(1\mathrm{P})(d)}_{FFBBB} =& \frac{1}{8}\int_0^\infty d\tau\, \langle \Big(Q\Phi_D\ \eta\Phi_E + \eta\Phi_D\ Q\Phi_E\Big) \\
&\qquad \times (\xi_c b_c)\ \Big(Q\Xi_A\ \eta\Psi_B - \eta\Psi_A\ Q\Xi_B\Big)\ \Phi_C\rangle_W, \qquad (31)
\end{aligned}
$$

$$
\mathcal{A}^{(1\mathrm{P})(e)}_{FFBBB} = \frac{1}{4}\int_0^\infty d\tau \, \Bigg( \langle \Phi_E \; \eta\Psi_A \; (\xi_c b_c) \; Q\Xi_B \; \Big( Q\Phi_C \; \eta\Phi_D - \eta\Phi_C \; Q\Phi_D \Big) \rangle_W
$$

$$
+ \langle \eta\Phi_E \; \Big( Q\Xi_A \; (\xi_c b_c) \; \eta\Psi_B - \eta\Psi_A \; (\xi_c b_c) \; Q\Xi_B \Big) \; Q(\Phi_C \; \Phi_D) \rangle_W \Bigg)
$$

$$
- \frac{1}{4}\langle \eta\Psi_A \; Q\Xi_B \; \Phi_C \; \Phi_D \; \Phi_E \rangle_W . \tag{32}
$$

No-propagator diagram :




$$
\mathcal{A}^{(\mathrm{NP})}_{FFBBB} = \frac{1}{12}\langle \Big( Q\Xi_A \; \eta\Psi_B + \eta\Psi_A \; Q\Xi_B \Big) \; \Phi_C \; \Phi_D \; \Phi_E \rangle_W . \tag{33}
$$

$$
\begin{aligned}
\mathcal{A}_{FFBBB} &= \sum_{i=a}^{e} \mathcal{A}^{(\mathrm{2P})(i)}_{FFBBB} + \sum_{i=a}^{e} \mathcal{A}^{(\mathrm{1P})(i)}_{FFBBB} + \mathcal{A}^{(\mathrm{NP})}_{FFBBB} \\
&= \int d^2\tau \, \Bigg( \langle \Big( Q\Xi_A \; \eta\Psi_B + \eta\Psi_A \; Q\Xi_B \Big) \; (\xi_{c_1} b_{c_1}) \; Q\Phi_C \; b_{c_2} \; Q\Phi_D \; \eta\Phi_E \rangle_W \\
&\qquad\qquad + \langle \eta\Psi_B \; Q\Phi_C \; (\xi_{c_1} b_{c_1}) \; Q\Phi_D \; b_{c_2} \; \eta\Phi_E \; Q\Xi_A \rangle_W \\
&\qquad\qquad + \langle Q\Xi_B \; Q\Phi_C \; (\xi_{c_1} b_{c_1}) \; Q\Phi_D \; b_{c_2} \; \eta\Phi_E \; \eta\Psi_A \rangle_W \Bigg) \\
&\qquad\qquad + \langle Q\Phi_C \; Q\Phi_D \; (\xi_{c_1} b_{c_1}) \; \eta\Phi_E \; b_{c_2} \; \Big( Q\Xi_A \; \eta\Psi_B + \eta\Psi_A \; Q\Xi_B \Big) \rangle_W \\
&\qquad\qquad + \langle Q\Phi_D \; \eta\Phi_E \; (\xi_{c_1} b_{c_1}) \; \Big( Q\Xi_A \; b_{c_2} \; \eta\Psi_B + \eta\Psi_A \; b_{c_2} \; Q\Xi_B \Big) \; Q\Phi_C \rangle_W \\
&\qquad\qquad + \langle \eta\Phi_E \; Q\Xi_A \; (\xi_{c_1} b_{c_1}) \; \eta\Psi_B \; b_{c_2} \; Q\Phi_C \; Q\Phi_D \rangle_W \\
&\qquad\qquad + \langle \eta\Phi_E \; \eta\Psi_A \; (\xi_{c_1} b_{c_1}) \; Q\Xi_B \; b_{c_2} \; Q\Phi_C \; Q\Phi_D \rangle_W \Bigg)
\end{aligned}
$$

$$
\begin{aligned}
= \int d^2\tau \Bigg( & \langle \eta\Psi_A \; \eta\Psi_B \; (\xi_{c_1} b_{c_1}) \; Q\Phi_C \; b_{c_2} \; Q\Phi_D \; \eta\Phi_E \rangle_W \\
& + \langle \eta\Psi_B \; Q\Phi_C \; (\xi_{c_1} b_{c_1}) \; Q\Phi_D \; b_{c_2} \; \eta\Phi_E \; \eta\Psi_A \rangle_W \\
& + \langle Q\Phi_C \; Q\Phi_D \; (\xi_{c_1} b_{c_1}) \; \eta\Phi_E \; b_{c_2} \; \eta\Psi_A \; \eta\Psi_B \rangle_W \\
& + \langle Q\Phi_D \; \eta\Phi_E \; (\xi_{c_1} b_{c_1}) \; \eta\Psi_A \; b_{c_2} \; \eta\Psi_B \; Q\Phi_C \rangle_W \\
& + \langle \eta\Phi_E \; \eta\Psi_A \; (\xi_{c_1} b_{c_1}) \; \eta\Psi_B \; b_{c_2} \; Q\Phi_C \; Q\Phi_D \rangle_W \Bigg),
\end{aligned}
\tag{34}
$$

after eliminating $\Xi$ by $Q\Xi = \eta\Psi$.

Since the final expression has the same form (up to $\xi_0$) as the *bosonic* SFT (with $\eta\Psi = V^{(-1/2)}, \; Q\Phi = V^{(0)} \longleftrightarrow A = V$), we can conclude that this is equal to the well-known amplitude.

Similarly, we can compute the ampolitudes with FFFFB and FBFBB, and show that the results are equal to the well-known amplitues.

## 4. Summary and discussion

♠ We have found that the missing gauge-symmetries are realized as those under which the variation of the pseudo-action is propotional to the constraint.

♠ We have proposed the new Feynman rules for the open SSFT respecting all the gauge- "symmetries".

♠ We have shown that the correct on-shell four- and five-point amplitudes are reproduced by the new rules.

♣ We can apply the similar argument to the heterotic string field theory, and obtain the new Feynmnan rules which reproduce the correct four- and five-point amplitudes without any ambiguity.

★ To clarify whether the new Feynman rules reproduce all the on-shell amplitudes at the tree level.

- general theory of the pseudo-action and its “symmetries”

★ To extend the Feynman rules to those applicable beyond the tree level.

- gauge fixing by means of the BV method

- extra factor $1/2$ for each fermion loop?