Closed Superstrings at Finite Temperature in the Framework of Thermo Field Dynamics

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Hokkaido Univ.

Kenji Hotta

1. Introduction

- Hagedorn Temperature $\mathcal{T}_H$ (type II)

  maximum temperature for perturbative strings

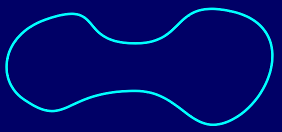
  A single energetic string captures most of the energy.

$$\beta_H \equiv \frac{1}{\mathcal{T}_H} = 2\pi\sqrt{2\alpha'}$$

$Z(\beta) \to \infty$ for $\beta < \beta_H$

The finite temperature systems of strings have been mainly investigated in Matsubara formalism.

- **Hagedorn Transition of Closed Strings**



$Z(\beta) \to \infty$ for $T > T_H$ (Matsubara Method)

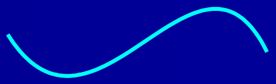
winding tachyon in the Euclidean time direction

Hagedorn Transition (Sathiapalan, Kogan, Atick-Witten)

A phase transition takes place due to the condensation of tachyon fields. (stable minimum?)

- **Brane-antibrane Pair Creation Transition**

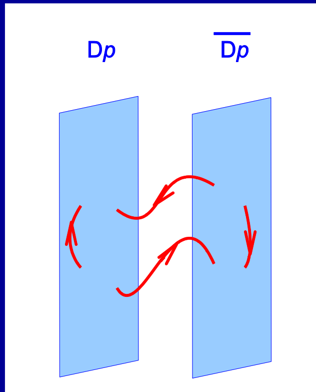
Dp-$\overline{\mathrm{D}p}$ Pairs are unstable at zero temperature



finite temperature system of Dp-$\overline{\mathrm{D}p}$ Pairs

→ D9-$\overline{\mathrm{D9}}$ pairs become stable near the Hagedorn temperature.

Hotta



- Relation between two phase transitions?

we have conjectured that

D9-$\overline{\text{D9}}$ Pairs are created
by the Hagedorn transition of closed strings.

These works are based on Matsubara Method.

One of the method to investigate finite temperature system is thermo field dynamics (TFD).

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finite temperature system of Dp-$\overline{\text{D}p}$ based on TFD

↓

finite temperature system of closed superstring based on TFD?

Thermo Field Dynamics (TFD) Takahashi-Umezawa

statistical average

$$\langle A\rangle = Z^{-1}(\beta)\sum_n \langle n|\hat{A}|n\rangle e^{-\beta E_n}$$

We can represent it as

$$\langle A\rangle = \left\langle 0(\beta)\left|\hat{A}\right|0(\beta)\right\rangle$$

by introducing a fictitious copy of the system.

$$|0(\beta)\rangle = Z^{-\frac{1}{2}}(\beta)\sum_n e^{-\frac{\beta E_n}{2}}\,|n\rangle\otimes|\tilde{n}\rangle$$ thermal vacuum state

The fictitious state is interpreted as `hole' state.

We cannot represent it as

$$|0(\beta)\rangle = \sum_n |n\rangle\, f_n(\beta)$$

for ordinary number $f_n(\beta)$, since

$$f_n^*(\beta) f_m(\beta) = Z^{-1}(\beta) e^{-\beta E_n}\delta_{nm}$$

cannot be satisfied.

Hawking-Unruh effect can be described by TFD.

It is expected that TFD is available to non-equilibrium system.

(real time formalism)

TFD has been applied to string theory

string field theory	Leblanc, Fujisaki, etc.
D-brane	Vancea, Cantcheff, etc.
closed bosonic string	Abdalla-Gadelha-Nedel
AdS background	Grada-Vancea, etc.
pp-wave background	Nedel-Abdalla-Gadelha, etc.

Unruh Effect in bosonic open string theory

Hata-Oda-Yahikozawa

→ closed string which can propagate bulk spacetime

finite temperature system of closed superstring

based on TFD?

Contents

2. Thermo Field Dynamics (Takahashi-Umezawa)

- Ensemble of Free Fermions (example)

  Hamiltonian $H = \omega a^\dagger a$

  anti-commutation relation $\left\{ a, a^\dagger \right\} = 1$

We introduce fictitious system.

  Hamiltonian $\tilde{H} = \omega \tilde{a}^\dagger \tilde{a}$

  anti-commutation relation $\left\{ \tilde{a}, \tilde{a}^\dagger \right\} = 1$

generator of Bogoliubov tr.

$$G_F = -i\theta(\beta) \left(\tilde{a}a - a^\dagger \tilde{a}^\dagger \right)$$

$$\begin{aligned} \sin\theta(\beta) &= \left(1 + e^{\beta\omega}\right)^{-\frac{1}{2}} \\ \cos\theta(\beta) &= \left(1 + e^{-\beta\omega}\right)^{-\frac{1}{2}} \\ \tan\theta(\beta) &= e^{-\frac{\beta\omega}{2}} \end{aligned}$$

thermal vacuum state

$$\begin{aligned} |0(\beta)\rangle &= e^{-iG_F}|0\rangle = \left\{\cos\theta(\beta) + \sin\theta(\beta) a^\dagger \tilde{a}^\dagger\right\}|0\rangle \\ &= \cos\theta(\beta)\exp\left[\tan\theta(\beta) a^\dagger \tilde{a}^\dagger\right]|0\rangle \end{aligned}$$

Bogoliubov tr. of annihilation ops.

$$\begin{aligned} a(\beta) &= e^{-iG_F} a e^{iG_F} = \cos\theta(\beta) a - \sin\theta(\beta)\tilde{a}^\dagger \\ \tilde{a}(\beta) &= e^{-iG_F}\tilde{a} e^{iG_F} = \cos\theta(\beta)\tilde{a} + \sin\theta(\beta) a^\dagger \end{aligned}$$

Thermal vacuum state satisfies

$$a(\beta)|0(\beta)\rangle = \tilde{a}(\beta)|0(\beta)\rangle = 0$$

Fermi distribution

$$\left\langle 0(\beta)\left|a^\dagger a\right|0(\beta)\right\rangle = \sin^2\theta(\beta) = \frac{e^{-\beta\omega}}{1+e^{-\beta\omega}}$$

fictitious system as `holes'

$$\frac{1}{\cos\theta(\beta)}\, a^\dagger|0(\beta)\rangle = -\frac{1}{\sin\theta(\beta)}\,\tilde{a}|0(\beta)\rangle$$

free energy

$$F(\theta) = \left\langle 0(\theta) \middle| \left(H - \frac{1}{\beta} K \right) \middle| 0(\theta) \right\rangle$$

entropy

$$K = -a^\dagger a \ln \sin^2\theta - a a^\dagger \ln \cos^2\theta$$

$$F(\theta) = \frac{1}{\beta} \ln \cos^2\theta + \left(\frac{1}{\beta} \ln \tan^2\theta + \omega \right) \sin^2\theta$$

relation between θ and β.

$$\frac{\partial}{\partial\theta} F(\theta) = 0 \quad \rightarrow \quad \tan^2\theta = e^{-\beta\omega}$$

$$F(\beta) = -\frac{1}{\beta} \ln\left(1 + e^{-\beta\omega}\right)$$

3. First Quantized Closed Superstring

- Light-Cone Momentum

We consider a single first quantized string.

light-cone momentum

$$p^0 = \frac{1}{\sqrt{2}}(p^+ + p^-)$$

$$2p^+p^- - |\boldsymbol{p}|^2 - M^2 = 0$$

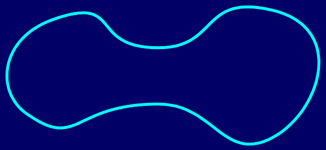
$$p^- = \frac{|\boldsymbol{p}|^2 + M^2}{2p^+}$$

$$\begin{aligned} p^+ &= \frac{1}{\sqrt{2}}\left(p^0 + p^1\right) \\ p^- &= \frac{1}{\sqrt{2}}\left(p^0 - p^1\right) \end{aligned}$$

partition function

$$\begin{aligned} Z_1(\beta) &= \mathrm{Tr}\ \exp\left(-\beta p^0\right) = \mathrm{Tr}\ \exp\left[-\frac{1}{\sqrt{2}}\,\beta(p^+ + p^-)\right] \\ &= \mathrm{Tr}\ \exp\left[-\frac{1}{\sqrt{2}}\,\beta\left(p^+ + \frac{|\boldsymbol{p}|^2 + M^2}{2p^+}\right)\right] \end{aligned}$$

Mass Spectrum



$$M_{NSNS}{}^2 = \frac{2}{\alpha'}\left(N_B + N_{NS} + \overline{N}_B + \overline{N}_{NS} - 1\right)$$

$$M_{RR}{}^2 = \frac{2}{\alpha'}\left(N_B + N_R + \overline{N}_B + \overline{N}_R\right)$$

space time boson

$$M_{NSR}{}^2 = \frac{2}{\alpha'}\left(N_B + N_{NS} + \overline{N}_B + \overline{N}_R - \frac{1}{2}\right)$$

$$M_{RNS}{}^2 = \frac{2}{\alpha'}\left(N_B + N_R + \overline{N}_B + \overline{N}_{NS} - \frac{1}{2}\right)$$

space time fermion

$$N_B = \sum_{l=1}^{\infty}\sum_{I=1}^{8} \alpha^I_{-l}\alpha^I_l, \quad N_{NS} = \sum_{r=\frac{1}{2}}^{\infty}\sum_{I=1}^{8} r b^I_{-r} b^I_r, \quad N_R = \sum_{m=1}^{\infty}\sum_{I=1}^{8} m d^I_{-m} d^I_m$$

We show only the NS-NS mode case.

GSO projection

$$\text{left-moving modes}: \ \frac{1}{2}(1+G)$$

$$\text{right-moving modes}: \ \frac{1}{2}\left(1+\overline{G}\right)$$

$$G = -(-1)^{\sum_{r=\frac{1}{2}}^{\infty} b_{-r} b_r}$$

level-matching condition

$$N_B + N_{NS} - \overline{N}_B - \overline{N}_{NS} = 0$$

$$\delta_{n,n'} = \int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \ \exp\left[2\pi i \tau_1 \left(n - n'\right)\right]$$

Thermal Vacuum State

generator of Bogoliubov tr.

$$G_{NSNS} = \mathcal{G}_B + \mathcal{G}_{NS} + \overline{\mathcal{G}}_B + \overline{\mathcal{G}}_{NS}$$

$$\mathcal{G}_B = i \sum_{l=1}^{\infty} \frac{1}{l} \theta_l \left(\alpha_{-l} \cdot \tilde{\alpha}_{-l} - \tilde{\alpha}_l \cdot \alpha_l \right)$$

$$\mathcal{G}_{NS} = i \sum_{r=\frac{1}{2}}^{\infty} \theta_r \left(b_{-r} \cdot \tilde{b}_{-r} - \tilde{b}_r \cdot b_r \right)$$

thermal vacuum state for a single string

$$\begin{aligned} |0_{1NSNS}(\theta)\rangle &\equiv e^{-iG_{1NSNS}}|0\rangle\rangle \left|p^+\right\rangle |\boldsymbol{p}\rangle \\ &= |0_B(\theta)\rangle\, |0_{NS}(\theta)\rangle \left|\overline{0}_B\left(\overline{\theta}\right)\right\rangle \left|\overline{0}_{NS}\left(\overline{\theta}\right)\right\rangle \left|p^+\right\rangle |\boldsymbol{p}\rangle \end{aligned}$$

$$|0_B(\theta)\rangle = \prod_{l=1}^{\infty} \left\{ \left(\frac{1}{\cosh(\theta_l)} \right)^8 \exp\left[\frac{1}{l} \tanh(\theta_l) \alpha_{-l} \cdot \tilde{\alpha}_{-l} \right] \right\} |0\rangle\rangle$$

$$|0_{NS}(\theta)\rangle = \prod_{r=\frac{1}{2}}^{\infty} (\cos(\theta_r))^8 \exp\left[\tan(\theta_r) b_{-r} \cdot \tilde{b}_{-r} \right] |0\rangle\rangle$$

Free Energy for a Single String

$$F^{IJ}_{1NSNS}(\theta) = \left\langle 0_{1NSNS}(\theta) \left| \left\{ H_{1NSNS} - \frac{1}{\beta}\left(K_{1NSNS} + C_{NSNS} + P^{I}_{NS} + \overline{P}^{J}_{NS} \right) \right\} \right| 0_{1NSNS}(\theta) \right\rangle$$

$$I, J = + \quad \longrightarrow \quad \frac{1}{2} \text{ part of GSO projection}$$

$$I, J = - \quad \longrightarrow \quad \frac{1}{2} G \text{ part of GSO projection}$$

Hamiltonian

$$H_{1NSNS} = \frac{1}{\sqrt{2}}\left(p^{+} + \frac{|\boldsymbol{p}|^2 + M^2_{NSNS}}{2p^{+}} \right)$$

entropy

$$\begin{aligned} K_{1NSNS} &= -\sum_{l=1}^{\infty} \frac{1}{l} \left\{ \alpha_{-l}\cdot\alpha_{l} \ln \sinh^2\theta_l - \alpha_l \cdot \alpha_{-l} \ln \cosh^2\theta_l \right\} \\ &\quad - \sum_{r=\frac{1}{2}}^{\infty} \left\{ b_{-r}\cdot b_r \ln \sin^2\theta_r + b_r \cdot b_{-r} \ln \cos^2 \theta_r \right\} \\ &\quad + (\text{right-movers}) \end{aligned}$$

level-matching condition

$$C_{NSNS} = 2\pi i \tau_1 \left(\sum_{l=1}^{\infty} \alpha_{-l}\cdot\alpha_l + \sum_{r=\frac{1}{2}}^{\infty} r b_{-r}\cdot b_r - \sum_{l'=1}^{\infty} \overline{\alpha}_{-l'}\cdot\overline{\alpha}_{l'} - \sum_{r'=\frac{1}{2}}^{\infty} r' \overline{b}_{-r'}\cdot \overline{b}_{r'} \right)$$

GSO projection

$$P^{+}_{NS} = \overline{P}^{+}_{NS} = 0$$

$$\begin{aligned} P^{-}_{NS} &= \pi i \left(\sum_{r=\frac{1}{2}}^{\infty} b_{-r}\cdot b_r + 1 \right) \\ \overline{P}^{-}_{NS} &= \pi i \left(\sum_{r=\frac{1}{2}}^{\infty} \overline{b}_{-r}\cdot \overline{b}_r + 1 \right) \end{aligned}$$

Relation between β and θ .

$$\frac{\partial}{\partial\theta_l} F^{IJ}_{1NSNS}(\theta)=0$$

$$\tanh\theta_l=\exp\left(-\frac{\beta l}{2\sqrt{2}\ \alpha' p^+}+\pi i\tau_1 l\right)$$

For $I=+$

$$\frac{\partial}{\partial\theta_r} F^{+J}_{1NSNS}(\theta)=0$$

$$\tan\theta_r=\exp\left(-\frac{\beta r}{2\sqrt{2}\ \alpha' p^+}+\pi i\tau_1 r\right)$$

For $I=-$

$$\frac{\partial}{\partial\theta_r} F^{-J}_{1NSNS}(\theta)=0$$

$$\tan\theta_r=\exp\left(-\frac{\beta r}{2\sqrt{2}\ \alpha' p^+}+\pi i\tau_1 r+\frac{\pi i}{2}\right)$$

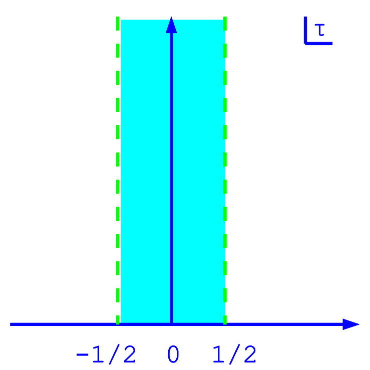
$$\begin{aligned}
F^{++}_{1NSNS}(\beta) \;=\; & \frac{1}{\sqrt{2}}\left(p^+ + \frac{|\boldsymbol{p}|^2}{2p^+}\right)+\frac{8}{\beta}\sum_{l=1}^{\infty}\ln\left[1-\exp\left(-\frac{\beta l}{\sqrt{2}\ \alpha' p^+}+2\pi i\tau_1 l\right)\right] \\
& -\frac{1}{2\sqrt{2}\ \alpha' p^+}+\frac{\pi i\tau_1}{\beta}-\frac{8}{\beta}\sum_{r=\frac{1}{2}}^{\infty}\ln\left[1+\exp\left(-\frac{\beta r}{\sqrt{2}\ \alpha' p^+}+2\pi i\tau_1 r\right)\right] \\
& +\frac{8}{\beta}\sum_{l'=1}^{\infty}\ln\left[1-\exp\left(-\frac{\beta l'}{\sqrt{2}\ \alpha' p^+}-2\pi i\tau_1 l'\right)\right] \\
& -\frac{1}{2\sqrt{2}\ \alpha' p^+}-\frac{\pi i\tau_1}{\beta}-\frac{8}{\beta}\sum_{r'=\frac{1}{2}}^{\infty}\ln\left[1+\exp\left(-\frac{\beta r'}{\sqrt{2}\ \alpha' p^+}-2\pi i\tau_1 r'\right)\right]
\end{aligned}$$

- Partition Function for a Single String

$$Z_{1NSNS}(\beta) = \frac{v_9}{2\sqrt{2}(2\pi)^9}\int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \int_0^\infty dp^+ \int_{-\infty}^{\infty} d^8p \times \left\{\exp\left(-\beta F_{1NSNS}^{++}\right) + \exp\left(-\beta F_{1NSNS}^{+-}\right) + \exp\left(-\beta F_{1NSNS}^{-+}\right) + \exp\left(-\beta F_{1NSNS}^{--}\right)\right\}$$

$$\tau_2 \equiv \frac{2\sqrt{2}\ \pi\beta}{{\beta_H}^2 p^+} \qquad \tau \equiv \tau_1 + i\tau_2 \qquad \beta_H = 2\pi\sqrt{2\alpha'}$$

$$Z_{1NSNS}(\beta) = \frac{8(2\pi)^8\beta v_9}{{\beta_H}^{10}}\int_S \frac{d^2\tau}{\tau_2^6}\frac{1}{|\vartheta_1{}'(0|\tau)|^8} \left\{\left(\vartheta_3^4 - \vartheta_4^4\right)\left(\bar\vartheta_3^4 - \bar\vartheta_4^4\right)\right\}(0|\tau)\exp\left(-\frac{2\pi\beta^2}{{\beta_H}^2\tau_2}\right)$$



domain of integration *S*

Free Energy for Multiple Strings

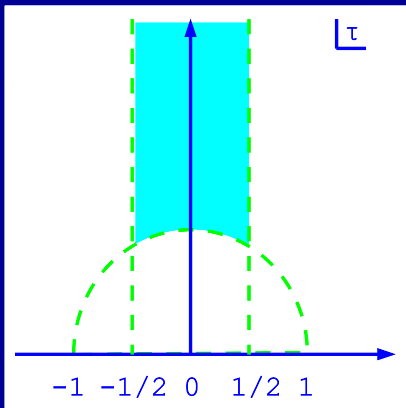
Free energy for multiple strings can be obtained from the following eq.

$$F(\beta) = -\sum_{w=1}^{\infty} \frac{1}{\beta w} [\{Z_{1NSNS}(\beta w) + Z_{1RR}(\beta w)\} - (-1)^w \{Z_{1NSR}(\beta w) + Z_{1RNS}(\beta w)\}]$$

$$\begin{aligned} F(\beta) \ = \ & -\frac{8(2\pi)^8 v_9}{\beta_H{}^{10}} \int_S \frac{d^2\tau}{\tau_2^6} \frac{1}{|\vartheta_1{}'(0|\tau)|^8} \\ & \times \left[\left\{ \left(\vartheta_3^4 - \vartheta_4^4\right)\left(\bar\vartheta_3^4 - \bar\vartheta_4^4\right) + \vartheta_2^4 \bar\vartheta_2^4 \right\}(0|\tau) \sum_{w=1}^{\infty} \exp\left(-\frac{2\pi w^2 \beta^2}{\beta_H{}^2 \tau_2}\right) \right. \\ & \left. - \left\{ \left(\vartheta_3^4 - \vartheta_4^4\right)\bar\vartheta_2^4 + \vartheta_2^4 \left(\bar\vartheta_3^4 - \bar\vartheta_4^4\right) \right\}(0|\tau) \sum_{w=1}^{\infty} (-1)^w \exp\left(-\frac{2\pi w^2 \beta^2}{\beta_H{}^2 \tau_2}\right) \right] \end{aligned}$$

This equals to the free energy in the S-representation based on Matsubara formalism.

We can transform this to the F-representation or the Dual-representation by using modular transformation.



4. Second Quantized Closed Superstring

- Light-Cone Gauge SFT cf) Kaku-Kikkawa, Hua

  We consider free string case.

  action

$$I_0 = \int_0^\infty dp^+ \int \mathcal{D}^8 X \; \left[\Phi^*_{p^+}(X,\psi) \; \left\{ i \, \frac{\partial}{\partial X^+} \; - \hat{p}^- \right\} \Phi_{p^+}(X,\psi) \right]$$

$$\hat{p}^- \;=\; \int_0^{2\pi p^+} d\sigma \sum_{I=2}^{9} \left\{ -\,\frac{\pi}{2}\,\frac{\delta^2}{\delta\left\{X^I(\sigma)\right\}^2} + \frac{1}{2\pi}\left(\frac{\partial X^I(\sigma)}{\partial\sigma}\right)^2 + 2i\left(\psi_L^I(\sigma)\,\frac{\partial}{\partial\sigma}\,\frac{\delta}{\delta\psi_L^I(\sigma)} - \psi_R^I(\sigma)\,\frac{\partial}{\partial\sigma}\,\frac{\delta}{\delta\psi_R^I(\sigma)}\right) \right\}$$

  eq. of motion

$$i\,\frac{\partial}{\partial X^+}\,\Phi_{p^+}(X,\psi) = \hat{p}^- \Phi_{p^+}(X,\psi)$$

$$\Phi_{NSNS}, \Phi_{RR}, \Phi_{NSR}, \Phi_{RNS}$$

  We show only the NS-NS mode case.

mode expansion

$$X^I(\sigma) = x^I + \sum_{l=1}^{\infty} \left\{ \frac{x_l^I}{\sqrt{l}} \cos\left(\frac{l\sigma}{p^+}\right) + \frac{y_l^I}{\sqrt{l}} \sin\left(\frac{l\sigma}{p^+}\right) \right\}$$

$$\psi_L^I = \sum_{r\in Z+\frac{1}{2}} \psi_r^I \exp\left(-\frac{ir\sigma}{p^+}\right)$$

$$\psi_R^I = \sum_{r\in Z+\frac{1}{2}} \overline{\psi}_r^I \exp\left(\frac{ir\sigma}{p^+}\right)$$

$$\hat{p}^- = \frac{1}{2p^+}\left\{ -\sum_{I=2}^{9} \frac{\partial^2}{\partial\left(x^I\right)^2} + \frac{2}{\alpha'}\left(\hat{B} + \hat{F} - 1\right) \right\}$$

$$\hat{B} = \sum_{l=1}^{\infty} \sum_{I=2}^{9} \frac{l}{2} \left\{ -\frac{\partial^2}{\partial\left(x_n^I\right)^2} - \frac{\partial^2}{\partial\left(y_n^I\right)^2} + \left(x_n^I\right)^2 + \left(y_n^I\right)^2 - 2 \right\}$$

$$\hat{F} = \sum_{r=\frac{1}{2}}^{\infty} \sum_{I=2}^{9} r \left(\psi_r^I \frac{\partial}{\partial \psi_r^I} + \overline{\psi}_r^I \frac{\partial}{\partial \overline{\psi}_r^I} \right)$$

solution

$$\Phi_{NSNS,\alpha} = \exp\left(i\boldsymbol{p}\cdot\boldsymbol{x} - ip_\alpha^- x^+\right) \overline{\phi}_{B,\alpha_1} \phi_{B,\alpha_2} \overline{\phi}_{NS,\alpha_3} \phi_{NS,\alpha_4}$$

$$\phi_{B,\alpha} = \prod_{l=1}^{\infty} \prod_{I=2}^{9} \frac{l^{\frac{1}{4}}}{2^{\frac{n}{2}} \pi^{\frac{1}{4}} \sqrt{(n_l^I)!}} H_{n_l^I}(x_l^I) \exp\left(-\frac{1}{2} {x_l^I}^2\right)$$

$$\phi_{NS,\alpha} = \prod_{r=\frac{1}{2}}^{\infty} \prod_{I=2}^{9} \left(\psi_r^I\right)^{n_r^I}$$

$$\alpha = \left\{ p^+, \boldsymbol{p}, N_B, N_{NS}, \overline{N}_B, \overline{N}_{NS} \right\}$$

second quantization

$$\Phi_{NSNS} = \sum_{\alpha} \left(A^{\dagger}_{NSNS,\alpha} \Phi^{*}_{NSNS,\alpha} + \Phi_{NSNS,\alpha} A_{NSNS,\alpha} \right)$$

commutation relation

$$\left[A_{NSNS,\alpha}, A^{\dagger}_{NSNS,\alpha'} \right] = \delta_{\alpha,\alpha'}$$

$$\int_0^{\infty} dp^+ \int D^{16}z \ \Phi^{*}_{NSNS} \ \hat{p}^{-} \Phi_{NSNS} = \sum_{\alpha} \frac{|\boldsymbol{p}|^2 + M^2_{NSNS}}{2p^+} \ A^{\dagger}_{NSNS,\alpha} A_{NSNS,\alpha}$$

$$D^{16}z = d^8x \prod_{l=1}^{\infty} d^8x_l d^8y_l \prod_{r=\frac{1}{2}}^{\infty} d\psi_r d\overline{\psi}_r$$

Hamiltonian

$$H_{NSNS} = \frac{1}{\sqrt{2}} \sum_{\alpha} \left(p^+ + \frac{|\boldsymbol{p}|^2 + M^2_{NSNS}}{2p^+} \right) \mathcal{P}_{NSNS,\alpha} P_{NSNS,\alpha} A^{\dagger}_{NSNS,\alpha} A_{NSNS,\alpha}$$

level-matching condition

$$\mathcal{P}_{NSNS,\alpha} = \int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \ \exp\left[2\pi i \tau_1 \left(N_B + N_{NS} - \overline{N}_B - \overline{N}_{NS} \right) \right]$$

GSO projection

$$P_{NSNS,\alpha} = \frac{1}{4} \left(1 + G_{n_r} \right) \left(1 + \overline{G}_{\overline{n}_r} \right)$$

$$G_{n_r} = -(-1)^{\sum_{r=\frac{1}{2}}^{\infty} n_r}$$

Thermal Vacuum State for NS-NS Strings

generator of Bogoliubov tr.

$$G_{NSNS} = i\sum_{\alpha} \theta_{NSNS,\alpha}\left(A^{\dagger}_{NSNS,\alpha}\tilde{A}^{\dagger}_{NSNS,\alpha} - \tilde{A}_{NSNS,\alpha}A_{NSNS,\alpha}\right)$$

$$\alpha = \{p^{+}, \boldsymbol{p}, N_B, N_{NS}, \overline{N}_B, \overline{N}_{NS}\}$$

thermal vacuum state for multiple strings

$$\begin{aligned}
|0_{NSNS}(\theta)\rangle &\equiv e^{-iG_{NSNS}}|0\rangle\rangle \\
&= \exp\left[\sum_{\alpha} \theta_{NSNS,\alpha}\left(A^{\dagger}_{NSNS,\alpha}\tilde{A}^{\dagger}_{NSNS,\alpha} - \tilde{A}_{NSNS,\alpha}A_{NSNS,\alpha}\right)\right]|0\rangle\rangle \\
&= \prod_{\alpha}\left\{\frac{1}{\cosh(\theta_{NSNS,\alpha})}\exp\left[\tanh(\theta_{NSNS,\alpha})A^{\dagger}_{NSNS,\alpha}\tilde{A}^{\dagger}_{NSNS,\alpha}\right]\right\}|0\rangle\rangle
\end{aligned}$$

$$A_{NSNS,\alpha}|0\rangle\rangle = \tilde{A}_{NSNS,\alpha}|0\rangle\rangle = 0$$

Free Energy for Multiple NS-NS String

$$F_{NSNS}(\theta) = \left\langle 0_{NSNS}(\theta) \middle| \left(H_{NSNS} - \frac{1}{\beta} K_{NSNS} \right) \middle| 0_{NSNS}(\theta) \right\rangle$$

Hamiltonian

$$H_{NSNS} = \frac{1}{\sqrt{2}} \sum_{\alpha} \left(p^{+} + \frac{|\boldsymbol{p}|^2 + M_{NSNS}^2}{2p^{+}} \right) \mathcal{P}_{NSNS,\alpha} P_{NSNS,\alpha} A^{\dagger}_{NSNS,\alpha} A_{NSNS,\alpha}$$

entropy

$$\begin{aligned} K_{NSNS} &= -\sum_{\alpha} \big(A^{\dagger}_{NSNS,\alpha} A_{NSNS,\alpha} \ln \sinh^2 \theta_{NSNS,\alpha} \\ &\qquad - A_{NSNS,\alpha} A^{\dagger}_{NSNS,\alpha} \ln \cosh^2 \theta_{NSNS,\alpha} \big) \mathcal{P}_{NSNS,\alpha} P_{NSNS,\alpha} \end{aligned}$$

$$\begin{aligned} F_{NSNS}(\theta) &= \sum_{\alpha} \mathcal{P}_{NSNS,\alpha} P_{NSNS,\alpha} \\ &\quad \times \left\{ \sinh^2 \theta_{NSNS,\alpha} \left(E_{NSNS,\alpha} + \frac{1}{\beta} \ln \tanh^2 \theta_{NSNS,\alpha} \right) \right. \\ &\qquad \left. - \frac{1}{\beta} \ln \cosh^2 \theta_{NSNS,\alpha} \right\} \end{aligned}$$

Relation between β and θ .

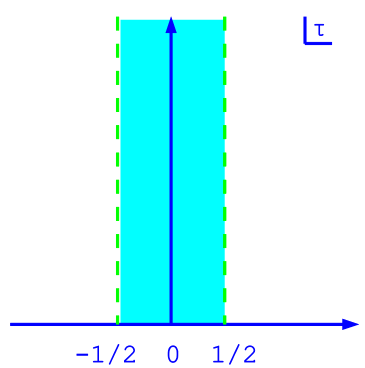
$$\frac{\partial}{\partial \theta_{NSNS,\alpha}} F_{NSNS}(\theta) = 0$$

$$\tanh \theta_{NSNS,\alpha} = \exp\left(-\frac{\beta E_{NSNS,\alpha}}{2}\right)$$

$$F_{NSNS}(\beta) = -\sum_{\alpha} \sum_{w=1}^{\infty} \frac{1}{w\beta} \mathcal{P}_{NSNS,\alpha} P_{NSNS,\alpha} \exp\left(-w\beta E_{NSNS,\alpha}\right)$$

$$\sum_{\alpha} \to \sum_{n_l,\bar{n}_l,n_r,\bar{n}_r} \frac{\sqrt{2}\, v_9}{(2\pi)^9} \int_0^{\infty} dp^+ \int_{-\infty}^{\infty} d^8p \qquad \tau_2 \equiv \frac{2\sqrt{2}\,\pi\beta}{{\beta_H}^2 p^+} \qquad \tau \equiv \tau_1 + i\tau_2 \qquad \beta_H = 2\pi\sqrt{2\alpha'}$$

$$F_{NSNS}(\beta) = -\frac{8(2\pi)^8 v_9}{\beta_H^{10}} \int_S \frac{d^2\tau}{\tau_2^6} \frac{1}{|\vartheta_1{}'(0|\tau)|^8} \times \left(\vartheta_3^4 - \vartheta_4^4\right)\left(\bar{\vartheta}_3^4 - \bar{\vartheta}_4^4\right)(0|\tau) \left\{\sum_{w=1}^{\infty} \exp\left(-\frac{2\pi w^2 \beta^2}{\beta_H^2 \tau_2}\right)\right\}$$



domain of integration *S*

Free Energy for Multiple Strings

Summing over the free energy for all sectors, we obtain

$$F(\beta) = F_{NSNS}(\beta) + F_{RR}(\beta) + F_{NSR}(\beta) + F_{RNS}(\beta)$$

$$\begin{aligned} F(\beta) \; = \; & -\frac{8(2\pi)^8 v_9}{{\beta_H}^{10}} \int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \int_0^{\infty} d\tau_2 \frac{1}{\tau_2^6} \frac{1}{|\vartheta_1{}'(0|\tau)|^8} \\ & \times \Bigg[\left\{ \left(\vartheta_3^4 - \vartheta_4^4\right)\left(\bar\vartheta_3^4 - \bar\vartheta_4^4\right) + \vartheta_2^4 \bar\vartheta_2^4 \right\}(0|\tau) \sum_{w=1}^{\infty} \exp\left(-\frac{2\pi w^2 \beta^2}{{\beta_H}^2 \tau_2}\right) \\ & - \left\{ \left(\vartheta_3^4 - \vartheta_4^4\right)\bar\vartheta_2^4 + \vartheta_2^4\left(\bar\vartheta_3^4 - \bar\vartheta_4^4\right) \right\}(0|\tau) \sum_{w=1}^{\infty} (-1)^w \exp\left(-\frac{2\pi w^2 \beta^2}{{\beta_H}^2 \tau_2}\right) \Bigg] \end{aligned}$$

This equals to the free energy in the S-representation based on Matsubara formalism.

We can transform this to the F-representation or Dual-representation by using modular transformation.

5. Conclusion and Discussion

- First Quantized Closed Superstring Case

  We computed thermal vacuum state and partition function
  for a single closed superstring based on TFD.

  The free energy for multiple strings agrees with that
  based on the Matsubara formalism.

  We need some gimmicks for calculation.

- Second Quantized Closed Superstring Case

  We computed thermal vacuum state and free energy
  for multiple closed superstrings based on TFD.

  The free energy for multiple strings agrees with that
  based on the Matsubara formalism.

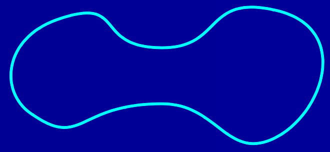
- D9-$\overline{\text{D9}}$ Pair Case

  We need to use second quantized string field theory in order to obtain the thermal vacuum state for multiple open strings.

- D-brane boundary state of closed string cf) Cantcheff

  The thermal vacuum state is reminiscent of the D-brane boundary state of a closed string.

$$|B9_{mat},\eta\rangle_{NSNS} = \exp\left[-\sum_{n=1}^{\infty}\frac{1}{n}\,\alpha_{-n}\cdot\tilde{\alpha}_{-n} + i\eta\sum_{u>0}\psi_{-u}\cdot\tilde{\psi}_{-u}\right]|B9_{mat},\eta\rangle^{(0)}_{NSNS}$$



- Hawking-Unruh Effect

  closed strings in curved spacetime

  Unruh Effect in bosonic open string theory

  Hata-Oda-Yahikozawa

  black hole firewall Almheiri-Marolf-Polchinski-Sully

  Planck solid model Hotta

A. Brane-antibrane Pair in TFD

- Light-Cone Momentum

We consider a single first quantized string.

partition function for a single string

$$Z_1(\beta) = \mathrm{Tr}\ \exp\left(-\beta p^0\right) = \mathrm{Tr}\ \exp\left[-\frac{1}{\sqrt{2}}\beta(p^+ + p^-)\right]$$
$$= \mathrm{Tr}\ \exp\left[-\frac{1}{\sqrt{2}}\beta\left(p^+ + \frac{|\boldsymbol{p}|^2 + M^2}{2p^+}\right)\right]$$

$$p^+ = \frac{1}{\sqrt{2}}\left(p^0 + p^1\right)$$
$$p^- = \frac{1}{\sqrt{2}}\left(p^0 - p^1\right)$$

- BSFT (Boundary String Field Theory) (BV formalism)

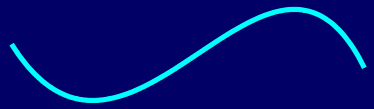
solution of classical master eq. (superstring)

$$S_{eff} = Z$$

S_{eff} : effective action Z : 2-dim. partition function

$$S_2 = \frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma\ \partial_a X_\mu \partial^a X^\mu + \int_{\partial\Sigma} d\tau |T|^2 + \cdots$$

Mass Spectrum



We consider an open string

on a Brane-antibrane pair.

mass spectrum

$$M_{NS}{}^2 = \frac{1}{\alpha'}\left(N_B + N_{NS} + 2|T|^2 - \frac{1}{2}\right)$$ space time boson

$$M_R{}^2 = \frac{1}{\alpha'}\left(N_B + N_R + 2|T|^2\right)$$ space time fermion

number ops.

$$N_B = \sum_{l=1}^{\infty}\sum_{I=1}^{8} \alpha_{-l}^I \alpha_l^I$$ oscillation mode of world sheet boson

$$N_{NS} = \sum_{r=\frac{1}{2}}^{\infty}\sum_{I=1}^{8} r b_{-r}^I b_r^I$$ oscillation mode of world sheet fermion (NS b. c)

$$N_R = \sum_{m=1}^{\infty}\sum_{I=1}^{8} m d_{-m}^I d_m^I$$ oscillation mode of world sheet fermion (R b. c)

We will show only the NS mode case.

Bogoliubov Transformation

generator of Bogoliubov tr.

$$G_{1NS} = \mathcal{G}_B + \mathcal{G}_{NS}$$

$$\mathcal{G}_B = i\sum_{l=1}^{\infty}\frac{1}{l}\,\theta_l\left(\alpha_{-l}\cdot\tilde{\alpha}_{-l}-\tilde{\alpha}_l\cdot\alpha_l\right)$$

$$\mathcal{G}_{NS} = i\sum_{r=\frac{1}{2}}^{\infty}\theta_r\left(b_{-r}\cdot\tilde{b}_{-r}-\tilde{b}_r\cdot b_r\right)$$

$$\tanh\theta_l = \exp\left(-\frac{\beta l}{4\sqrt{2}\alpha' p^+}\right)$$

$$\tan\theta_r = \exp\left(-\frac{\beta r}{4\sqrt{2}\alpha' p^+}\right)$$

Thermal Vacuum State

thermal vacuum state for a single string

$$|0_{1NS}(\theta)\rangle \equiv e^{-iG_{1NS}}|0\rangle\rangle\left|p^+\right\rangle|\boldsymbol{p}\rangle$$

$$= \prod_{l=1}^{\infty}\left\{\left(\frac{1}{\cosh(\theta_l)}\right)^8\exp\left[\frac{1}{l}\tanh(\theta_l)\alpha_{-l}\cdot\tilde{\alpha}_{-l}\right]\right\}$$

$$\times\prod_{r=\frac{1}{2}}^{\infty}\left\{(\cos(\theta_r))^8\exp\left[\tan(\theta_r)b_{-r}\cdot\tilde{b}_{-r}\right]\right\}|0\rangle\rangle\left|p^+\right\rangle|\boldsymbol{p}\rangle$$

$$\alpha_l|0\rangle\rangle = b_r|0\rangle\rangle = \tilde{\alpha}_l|0\rangle\rangle = \tilde{b}_r|0\rangle\rangle = 0 \qquad \text{for positive } l, r$$

Free Energy for a Single String

$$F_{1NS}(\theta)=\left\langle 0_{1NS}(\theta)\right|\left(H_{1NS}-\frac{1}{\beta}K_{1NS}\right)\left|0_{1NS}(\theta)\right\rangle$$

Hamiltonian

$$H_{1NS}=\frac{1}{\sqrt{2}}\left(p^{+}+\frac{|\boldsymbol{p}|^2+M_{NS}^2}{2p^{+}}\right)$$

entropy

$$K_{1NS} = -\sum_{l=1}^{\infty}\frac{1}{l}\left\{\alpha_{-l}\cdot\alpha_l \ln\sinh^2\theta_l-\alpha_l\cdot\alpha_{-l}\ln\cosh^2\theta_l\right\} -\sum_{r=\frac{1}{2}}^{\infty}\left\{b_{-r}\cdot b_r\ln\sin^2\theta_r+b_r\cdot b_{-r}\ln\cos^2\theta_r\right\}$$

$$F_{1NS}(\beta) = \frac{1}{\sqrt{2}}\left(p^{+}+\frac{|\boldsymbol{p}|^2}{2p^{+}}\right)+\frac{|T|^2}{\sqrt{2}\alpha' p^{+}}+\frac{8}{\beta}\sum_{l=1}^{\infty}\ln\left[1-\exp\left(-\frac{\beta l}{2\sqrt{2}\alpha' p^{+}}\right)\right] -\frac{1}{4\sqrt{2}\alpha' p^{+}}-\frac{8}{\beta}\sum_{r=\frac{1}{2}}^{\infty}\ln\left[1+\exp\left(-\frac{\beta r}{2\sqrt{2}\alpha' p^{+}}\right)\right]$$

This is not useful for analysis of thermodynamical system of strings.

free energy for a single string

➡ partition function for a single string ➡ free energy for multiple strings (string gas)

Partition Function for a Single String

$$Z_{1NS}(\beta) = \frac{v_p}{(2\pi)^p}\int_0^\infty dp^+ \int_{-\infty}^{\infty} d^p p \;\; \exp\left(-\beta F_{1NS}\right)$$

$$\tau \equiv \frac{2\pi\beta}{{\beta_H}^2 p^+} = \frac{\beta}{4\pi\alpha' p^+}, \quad \beta_H \;=\; 2\pi\sqrt{2\alpha'}$$

$$Z_{1NS}(\beta) \;=\; \frac{16\pi^4\beta v_p}{{\beta_H}^{p+1}}\int_0^\infty \frac{d\tau}{\tau}\;\tau^{-\frac{p+1}{2}} e^{-4\pi|T|^2}\left\{\frac{\vartheta_3(0|i\tau)}{{\vartheta_1}'(0|i\tau)}\right\}^4 \exp\left(-\frac{\pi\beta^2}{{\beta_H}^2\tau}\right)$$

Free Energy for Multiple Strings

Free energy for multiple strings can be obtained from the following eq.

$$F(\beta) = -\sum_{w=1}^{\infty}\frac{1}{\beta w}\left\{Z_{1NS}(\beta w) - (-1)^w Z_{1R}(\beta w)\right\}$$

$$F(\beta) \;=\; -\frac{16\pi^4 v_p}{{\beta_H}^{p+1}}\int_0^\infty \frac{d\tau}{\tau}\;\tau^{-\frac{p+1}{2}} e^{-4\pi|T|^2\tau} \times\left[\left(\frac{\vartheta_3(0|i\tau)}{{\vartheta_1}'(0|i\tau)}\right)^4\left\{\vartheta_3\left(0\left|\frac{i\beta^2}{{\beta_H}^2\tau}\right.\right)-1\right\} - \left(\frac{\vartheta_2(0|i\tau)}{{\vartheta_1}'(0|i\tau)}\right)^4\left\{\vartheta_4\left(0\left|\frac{i\beta^2}{{\beta_H}^2\tau}\right.\right)-1\right\}\right]$$

This equals to the free energy based on Matsubara formalism.

This implies that our choice of Weyl factors in the case of Matsubara formalism is quite natural.