

Partial Breaking of N=2 Supersymmetry and Low Energy Theorem

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based on collaboration with Hiroshi Itoyama, Syoji Minato
and [arXiv:0704.1060](https://arxiv.org/abs/0704.1060); [arXiv:0710.4377](https://arxiv.org/abs/0710.4377); [arXiv:0909.5486](https://arxiv.org/abs/0909.5486)

1. Introduction

Supersymmetry is one of the most attractive ideas in theoretical physics. One of the advantages of supersymmetry on which we focus is its *highly symmetry*. By symmetry, we fortunately can evaluate the low energy effective action in some cases.

Various investigations have been made on the low energy effective action of supersymmetric gauge theory. For example, now we can compute

- the low energy effective action of $N=2$ supersymmetric gauge theory
- the low energy effective superpotential of $N=1$ supersymmetric gauge theory.

In particular, $N=1$ supersymmetric gauge theories are more realistic, because of the vacuum structure and other various reasons.

In this talk, we consider $N=1$ (and $N=2$), supersymmetric $U(N)$ gauge theory whose matter content is

vector multiplet

$$(A_\mu, \lambda^\alpha)$$

adjoint chiral multiplet

$$(\phi, \psi^\alpha)$$

In 2002, Dijkgraaf and Vafa have suggested that the effective superpotential of $N=1$, $U(N)$ gauge theory with superpotential

$$\int d^2\theta \text{Tr}(\tau_{\text{cl}} \mathcal{W}^\alpha \mathcal{W}_\alpha + W_{\text{tree}}), \quad W_{\text{tree}}(\Phi) = \sum_{k=1}^{n+1} \frac{g_k}{k} \Phi^k$$

can be written in terms of the free energy of the bosonic one matrix model by the following formula: [Dijkgraaf-Vafa]

$$W_{\text{eff}}(S) = N \frac{\partial}{\partial S} F_{\text{free}}(S) \quad S = -\frac{1}{64\pi^2} \text{Tr}_{\text{SU}(N)} \mathcal{W}^\alpha \mathcal{W}_\alpha$$

U(N) Partial breaking model [Fujiwara-Itoyama-Sakaguchi '04,'05]

The model we focus on is as follows:

$$\int d^2\theta \text{Tr} \left(\mathcal{F}''(\Phi) \mathcal{W}^\alpha \mathcal{W}_\alpha + e\Phi + m \frac{\partial \mathcal{F}(\Phi)}{\partial \Phi} \right)$$

This Lagrangian has N=2 supersymmetry. However, the vacua preserves only N=1 supersymmetry.

N=2 supersymmetry is partially broken to N=1

There exists a massless fermion which is Nambu-Goldstone particle.

What happen at the low energy?

1. The effective superpotential is deformed compared with that of Dijkgraaf-Vafa:

$$W_{\text{eff}}(S) = N \frac{\partial F_{\text{free}}(S)}{\partial S} + \frac{16\pi^2 i}{m} \sum_{k=2}^{n+1} g_k \frac{\partial F_{\text{free}}(S)}{\partial g_{k-1}}$$

2. We can find the low energy theorem for the Nambu-Goldstone fermion.

For example, the amplitude $\bar{u}(q)M(q)$ for the process

$$A \rightarrow \lambda_{\text{NG}}(q) + B$$

is suppressed:

$$\lim_{q \rightarrow 0} M(q) = 0$$

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2. U(N) gauge model with partially broken N=2 supersymmetry

First of all, due to **modification of the supersymmetry current algebra**, it is possible to break N=2 supersymmetry partially:

$$\{\bar{Q}_{\dot{\alpha}}^j, J_{\alpha i}^{\mu}\} = 2(\sigma^{\nu})_{\alpha\dot{\alpha}}\delta_i^j T_{\nu}^{\mu} + (\sigma^{\mu})_{\alpha\dot{\alpha}} C_i^j \leftarrow \text{constants}$$

N=1 superspace formalism

[hep-th/0409060, 0503113]

N=2 harmonic superspace formalism

[hep-th/0510255]

by Fujiwara, Itoyama and Sakaguchi

(N=2 quiver gauge theory [Itoyama-K.M.-Sakaguchi, arXiv:0709.3166])

In the following, we construct N=2, U(N) gauged model in N=1 superspace formalism.

gauge index: $a = 0, 1, \dots, N^2 - 1$

U(N) vector superfield

$$V^a = (A_{\mu}^a, \lambda^a)$$

overall U(1)

U(N) adjoint chiral superfield

$$\Phi^a = (\phi^a, \psi^a)$$

Generic action with U(N) isometry consists of

- Kahler term

$$S_K = \frac{-i}{2} \int d^4x d^4\theta \text{Tr} \left(\bar{\Phi} e^{adV} \frac{\partial \mathcal{F}(\Phi)}{\partial \Phi} \right) + h.c.,$$

prepotential
↓

- gauge kinetic term

$$S_{\text{gauge}} = \frac{-i}{4} \int d^4x d^2\theta \tau_{ab}(\Phi) \mathcal{W}^{\alpha a} \mathcal{W}_{\alpha}^b + h.c.,$$

- superpotential

↑
function of Φ

$$S_W = \int d^4x d^2\theta \text{Tr} W(\Phi) + h.c.,$$

By imposing discrete R invariance:

$$R : \begin{pmatrix} \lambda^a \\ \psi^a \end{pmatrix} \rightarrow \begin{pmatrix} \psi^a \\ -\lambda^a \end{pmatrix},$$

the generalized gauge coupling $\tau_{ab}(\Phi)$ and the superpotential take the following forms,

$$\tau_{ab}(\Phi) = \frac{\partial^2 \mathcal{F}(\Phi)}{\partial \Phi^a \partial \Phi^b}, \quad W(\Phi) = e\Phi^0 + m \frac{\partial \mathcal{F}(\Phi)}{\partial \Phi^0}.$$

e : complex, m : real

➡ $\mathcal{N}=2$ supersymmetry invariant action is

$$S = \int d^4x d^4\theta \frac{-i}{2} \text{Tr} \left(\bar{\Phi} e^{adV} \frac{\partial \mathcal{F}(\Phi)}{\partial \Phi} - h.c. \right) \\ + \int d^4x d^2\theta \left(-\frac{i}{4} \frac{\partial^2 \mathcal{F}(\Phi)}{\partial \Phi^a \partial \Phi^b} \mathcal{W}^{\alpha a} \mathcal{W}_\alpha^b + e\Phi^0 + m \frac{\partial \mathcal{F}(\Phi)}{\partial \Phi^0} \right) + h.c.$$

Vacuum structure


In the following, we consider the case of

$$\mathcal{F}(\Phi) = \sum_{k=0}^{n+1} \frac{g_k}{(k+1)!} \text{Tr} \Phi^{k+1} \quad ; \text{ degree } n+2.$$

Scalar potential

$$V = g^{ab} \left(\frac{1}{8} \mathcal{D}_a \mathcal{D}_b + \partial_a \left(e\phi^0 + m \frac{\partial \mathcal{F}(\phi)}{\partial \phi^0} \right) \overline{\partial_b \left(e\phi^0 + m \frac{\partial \mathcal{F}(\phi)}{\partial \phi^0} \right)} \right)$$

Vacuum condition $\frac{\partial V}{\partial \phi^a} = 0$ leads to

 $g^{bd} \mathcal{F}_{ade} g^{ec} (e\delta_b^0 + m\bar{\mathcal{F}}_{0b}) (\bar{e}\delta_c^0 + m\bar{\mathcal{F}}_{0c}) = 0$

Change the basis for Cartan part (eigenvalue basis)

$$(t_{\underline{i}})^k_{\ell} = \delta_{\underline{i}}^k \delta_{\ell}^{\underline{i}}, \quad (\underline{i} = 1, \dots, N, \quad j, k = 1, \dots, N)$$

In this basis, it is straightforward to examine the vacuum condition:

$$0 = \underbrace{\langle \mathcal{F}_{\underline{ij}} (g^{\underline{ij}})^2 \rangle}_{=0} \underbrace{\langle (2e + m\bar{\mathcal{F}}_{\underline{ij}})(2\bar{e} + m\bar{\mathcal{F}}_{\underline{ij}}) \rangle}_{=0}, \quad \underline{i} \text{ not summed.}$$

unstable vacua

stable vacua

Thus, the vacua are determined by

$$\langle \mathcal{F}_{\underline{ij}} \rangle = -\frac{2e}{m} \quad \text{or} \quad -\frac{2\bar{e}}{m}, \quad \text{for each } \underline{i} (= 1, \dots, N),$$

or, in terms of Kahler metric $g_{\underline{ij}} = \text{Im}\mathcal{F}_{\underline{ij}}$, $\langle g_{\underline{ij}} \rangle$ are gauge couplings $\left(\frac{1}{g^2}\right)_{\underline{i}}$

$$\langle g_{\underline{ij}} \rangle = -\frac{2 \text{Im}e}{m} \quad \text{or} \quad +\frac{2 \text{Im}e}{m}, \quad \text{for each } \underline{i} (= 1, \dots, N).$$

When $\text{Im}e/m < 0$,

- vacua A: $\langle g_{\underline{i}\underline{i}} \rangle = -2 \text{Im}e/m > 0$, for all \underline{i} . In this case, as we will see, the 1st (manifest) N=1 SUSY is preserved. Also, this condition determines the vev of ϕ , which, in general, breaks the gauge symmetry as

$$U(N) \rightarrow \prod_{i=1}^n U(N_i)$$

- vacua B: $\langle g_{\underline{i}\underline{i}} \rangle = 2 \text{Im}e/m < 0$, for all \underline{i} .
2nd (hidden) N=1 SUSY is preserved in these vacua.

Note that we have ignored the other vacua where $\langle g_{\underline{i}\underline{i}} \rangle$ are equal to $2 \text{Im}e/m$ for some \underline{i} and to $-2 \text{Im}e/m$ for the other \underline{i} .

Nambu-Goldstone fermion

The supersymmetry transformation law of the fermions is as follows:

$$\delta \begin{pmatrix} \lambda^a \\ \psi^a \end{pmatrix} = -\sqrt{2}g^{ab} \begin{pmatrix} 0 & -(e\delta_b^0 + m\bar{\mathcal{F}}_{0b}) \\ \bar{e}\delta_b^0 + m\bar{\mathcal{F}}_{0b} & 0 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} + \dots,$$

In the vacua A, this becomes

$$\langle \delta\psi^a \rangle = 0, \quad \langle \delta\lambda^a \rangle = 2\sqrt{2}m\eta_2\delta^a_0 \quad \longrightarrow \quad 2^{\text{nd}} \text{ SUSY is broken}$$

In the vacua B,

$$\langle \delta\psi^a \rangle = 2\sqrt{2}m\eta_1\delta^a_0, \quad \langle \delta\lambda^a \rangle = 0 \quad \longrightarrow \quad 1^{\text{st}} \text{ SUSY is broken}$$

mass spectrum

Nambu-Goldstone fermion !

- ◆ N=1 massless $\prod_{i=1}^n U(N_i)$ vector multiplets: $V^a = (A_\mu^a, \lambda^a \text{ (or } \psi^a))$
- ◆ N=1 massive $\prod_{i=1}^n U(N_i)$ adjoint chiral multiplets: $\tilde{\Phi}^a = (\tilde{\phi}^a, \psi^a \text{ (or } \lambda^a))$
- ◆ N=1 massive vector multiplets corresponding to broken generators

3. Effective superpotential

References: Hiroshi Itoyama and K.M., arXiv:0704.1060; 0710.4377

Suppose that each $U(N_i)$ gauge part confines at low energy. The effective superpotential can be written by gluino condensate superfield S_i .

$$(S_i = -\frac{1}{64\pi^2} \langle \text{Tr}_{U(N_i)} \mathcal{W}^{i\alpha} \mathcal{W}_\alpha^i \rangle)$$

It is well-known that this type of effective superpotential can be computed from **the bosonic one matrix model**. [Dijkgraaf-Vafa]

In the model we consider here, the effective superpotential can be also written by the language of the matrix model. **However there exists the deformation part in addition to well-known DV part.**

[Itoyama-K.M., Ferrari]
[Aganagic-Beem-Seo-Vafa]

$$W_{\text{eff}}(S) = N_i \frac{\partial F_{\text{free}}(S)}{\partial S_i} + \frac{16\pi^2 i^{n+1}}{m} \sum_{k=2}^{n+1} g_k \frac{\partial F_{\text{free}}(S)}{\partial g_{k-1}}$$

One matrix model (M is $\hat{N} \times \hat{N}$ matrix)

- definition:

$$\exp\left(-\frac{\hat{N}^2}{g_m^2} F_{\text{free}}\right) = \int dM \exp\left(-\frac{\hat{N}}{g_m} \text{tr} W(M)\right), \quad W(M) = \sum_{\ell=2}^{n+1} \frac{m g_k}{\ell!} M^\ell$$

- loop equation: In the large \hat{N} limit, loop equation becomes

$$R_m(z)^2 = W'(z)R_m(z) + \frac{f_m(z)}{4}, \quad R_m(z) \equiv \frac{g_m}{\hat{N}} \left\langle \text{tr} \frac{1}{z - M} \right\rangle$$

where $f_m(z)$ (of degree n-1) is determined by

$$g_m \frac{\hat{N}_i}{\hat{N}} = \oint_{\alpha_i} dz R_m(z)$$

- formula: Finally, by taking a variational derivative with respect to g_k , we obtain

$$\frac{\partial F_{\text{free}}}{\partial g_k} = \frac{g_m}{\hat{N}} \left\langle \frac{m}{k!} \text{tr} M^k \right\rangle.$$

Generalized Konishi anomaly equations and Chiral ring

[Itoyama-K.M. arXiv:0704.1060; 0710.4377]

cf) [Cachazo-Douglas-Seiberg-Witten; Ferrari, arXiv:0709.0472]

Recall that the action of our model is

$$S = \int d^4x d^2\theta \left(-\frac{i}{4} \frac{\partial^2 \mathcal{F}(\Phi)}{\partial \Phi^a \partial \Phi^b} \mathcal{W}^{\alpha a} \mathcal{W}_\alpha^b + e \Phi^0 + m \frac{\partial \mathcal{F}(\Phi)}{\partial \Phi^0} \right) + h.c. + \dots$$

and the matter induced part of the effective superpotential is defined by

$$\exp \left[i \int d^4x (d^2\theta W_{\text{eff}} + h.c. + d^4\theta (\text{nonchiral terms})) \right] = \int \mathcal{D}\Phi \mathcal{D}\bar{\Phi} e^{iS}.$$

The Schwinger-Dyson equation for general transformations $\delta\Phi = f(\Phi, \mathcal{W}^\alpha)$ is

$$\underbrace{\langle \bar{D}^2 J \rangle}_{\parallel 0} = \left\langle \frac{1}{64\pi^2} \left[\mathcal{W}^\alpha, \left[\mathcal{W}_\alpha, \frac{\partial f}{\partial \Phi_{ij}} \right] \right]_{ij} \right\rangle - \left\langle \frac{i}{4} \text{Tr} (f \mathcal{F}'''(\Phi) \mathcal{W}^\alpha \mathcal{W}_\alpha) \right\rangle + \langle \text{Tr} (f W'(\Phi)) \rangle.$$

Let us define the generating functions of the one-point functions:

$$R(z) = -\frac{1}{64\pi^2} \text{Tr} \left\langle \mathcal{W}^\alpha \mathcal{W}_\alpha \frac{1}{z - \Phi} \right\rangle, \quad T(z) = \text{Tr} \left\langle \frac{1}{z - \Phi} \right\rangle.$$

For $f(\Phi, \mathcal{W}) = -\frac{1}{64\pi^2} \mathcal{W}^\alpha \mathcal{W}_\alpha \frac{1}{z - \Phi}$ and $\frac{1}{z - \Phi}$, we get the following equations:

$$R(z)^2 = W'(z)R(z) + \frac{f(z)}{4}, \quad S_i = \oint_{\alpha_i} R(z) dz$$

where $f(z)$ and $c(z)$ are polynomial in z of degree $n-1$.
From the form of the equation for $R(z)$,

if $S_i = g_m \hat{N}_i / \hat{N}$, then $R(z) = R_m(z)$.

The formula for the free energy becomes

$$\frac{\partial F_{\text{free}}}{\partial g_k} = \frac{g_m}{\hat{N}} \left\langle \frac{m}{k!} \text{tr} M^k \right\rangle = \frac{m}{k!} v_k, \quad v_k \equiv -\frac{\left\langle \text{Tr} \Phi^{k-1} \mathcal{W}^\alpha \mathcal{W}_\alpha \right\rangle}{64\pi^2}.$$

The solutions of the above equations are

$$\left\{ \begin{array}{l} R(z) = \frac{1}{2} \left(W'(z) - \sqrt{W'^2(z) + f(z)} \right), \\ T(z) = -\frac{c(z)}{\sqrt{W'^2(z) + f(z)}} + 8\pi^2 i \left(\mathcal{F}'''(z) - \frac{W'(z)\mathcal{F}'''(z)}{\sqrt{W'^2(z) + f(z)}} \right). \end{array} \right.$$

Note that $T(z)$ satisfies $N_i = \frac{1}{2\pi i} \oint_{\alpha_i} T(z) dz$.

If we take this into account, above solution for $T(z)$ can be written, after some algebra, as

$$u_k = \sum_i N_i \frac{\partial v_k}{\partial S_i} + \frac{16\pi^2 i^{n+1}}{m} \sum_{\ell=2} g_\ell \frac{\partial v_k}{\partial g_{\ell-1}}, \quad \text{for all } k$$

where

$$u_k \equiv \langle \text{Tr} \Phi^k \rangle, \quad v_k \equiv -\frac{\langle \text{Tr} \Phi^{k-1} \mathcal{W}^\alpha \mathcal{W}_\alpha \rangle}{64\pi^2}.$$

Recall that the definition of the effective superpotential is

$$\exp \left[i \int d^4x (d^2\theta W_{\text{eff}} + h.c. + d^4\theta (\text{nonchiral terms})) \right] = \int \mathcal{D}\Phi \mathcal{D}\bar{\Phi} e^{iS}.$$

Final step is to take variational derivative of above with respect to the coupling g_k , we obtain

$$\begin{aligned} \frac{\partial W_{\text{eff}}}{\partial g_k} &= \frac{m}{k!} u_k + \frac{16\pi^2 i}{(k-1)!} v_{k-1} \\ &= \frac{m}{k!} \sum_i N_i \frac{\partial v_k}{\partial S_i} + \frac{16\pi^2 i}{(k-1)!} v_{k-1} + \frac{16\pi^2 i}{k!} \sum_{\ell=2}^{n+1} g_\ell \frac{\partial v_k}{\partial g_{\ell-1}}. \\ &= \frac{\partial}{\partial g_k} \left[\sum_i N_i \frac{\partial F_{\text{free}}}{\partial S_i} + \frac{16\pi^2 i}{m} \sum_{\ell=2}^{n+1} g_\ell \frac{\partial F_{\text{free}}}{\partial g_{\ell-1}} \right] \end{aligned}$$

Thus, g_k dependent part of the effective superpotential can be written as

$$W_{\text{eff}} = \sum_i N_i \frac{\partial F_{\text{free}}}{\partial S_i} + \frac{16\pi^2 i}{m} \sum_{\ell=2}^{n+1} g_\ell \frac{\partial F_{\text{free}}}{\partial g_{\ell-1}}$$

4. Low energy theorem for Nambu-Goldstone fermion

Reference: Hiroshi Itoyama, K.M and Syoji Minato, arXiv:0909.5486

The supercurrent associated with the broken N=1 supersymmetry is

$$\begin{aligned} \eta_2 \mathcal{S}^{(2)\mu} = & \frac{i}{2} g_{ab} \eta_2 \sigma^\rho \bar{\sigma}^\nu \sigma^\mu \bar{\psi}^a F_{\nu\rho}^b + 2i\sqrt{N} (e\delta_a^0 + m\bar{\mathcal{F}}_{0a}) \eta_2 \sigma^\mu \bar{\lambda}^a - \sqrt{2} g_{ab} \eta_2 \sigma^\nu \bar{\sigma}^\mu \lambda^a \mathcal{D}_\nu \bar{\phi}^b \\ & - \frac{i}{2} g_{ab} f_{cd}^b \eta_2 \sigma^\mu \bar{\psi}^a \bar{\phi}^c \phi^d + \frac{1}{2\sqrt{2}} \mathcal{F}_{abc} \eta_2 \lambda^a (\lambda^b \sigma^\mu \bar{\lambda}^c) + \frac{1}{2\sqrt{2}} \bar{\mathcal{F}}_{abc} \eta_2 \sigma^\mu \bar{\lambda}^a (\psi^b \bar{\psi}^c). \end{aligned}$$

In the N=1 vacua, this can be written as

$$\eta_2 \mathcal{S}^{(2)\mu} = \frac{i}{2} \langle g_{ab} \rangle \eta_2 \sigma^\rho \bar{\sigma}^\nu \sigma^\mu \bar{\psi}^a F_{\nu\rho}^b + 2i\sqrt{N} \langle e\delta_a^0 + m\bar{\mathcal{F}}_{0a} \rangle \eta_2 \sigma^\mu \bar{\lambda}^a + \dots$$

Next, we will consider the matrix element of this current:

$$\langle B_1, B_2, \dots, B_n | \mathcal{S}^{(2)\mu} | A_1, A_2, \dots, A_m \rangle$$

Process 1 $A \rightarrow \lambda_{\text{NG}}(q) + B$: amplitude $M_1(q)$.

For this process we consider the matrix element:

$$\langle B | \mathcal{S}^{(2)\mu} | A \rangle$$

Diagrammatically, this is depicted as

$$= \langle 0 | \mathcal{S}^\mu | \lambda \rangle \times \frac{i q^\mu \gamma_\mu}{q^2} \times M_1(q) \quad + \quad (\text{non-singular})$$

Thus, the momentum conservation $\partial_\mu \langle B | \mathcal{S}^{(2)\mu} | A \rangle = 0$ leads to

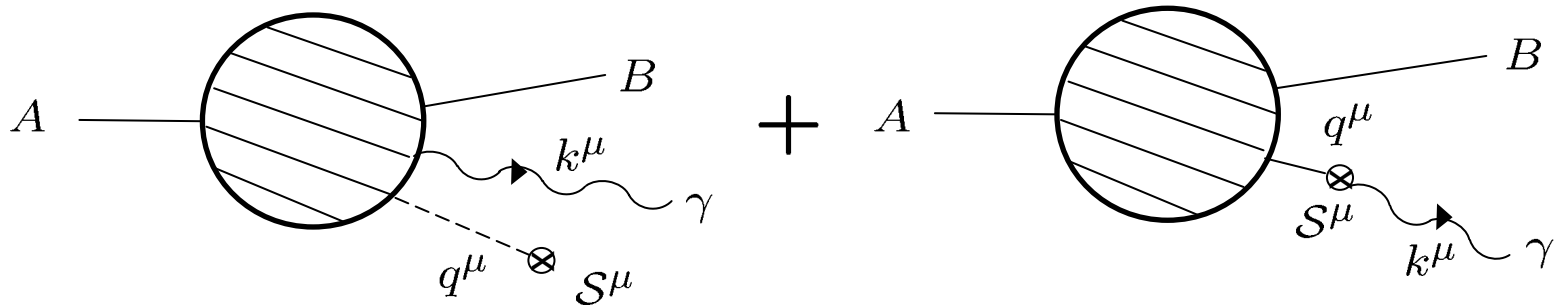
$$\lim_{q^\mu \rightarrow 0} M_1 = 0.$$

Process 2 $A \rightarrow \lambda_{\text{NG}}(q) + \gamma(k) + B$: amplitude $M_2(k, q)$

For this process we consider the matrix element:

$$\langle B, \gamma | \mathcal{S}^{(2)\mu} | A \rangle$$

The diagrams which could be singular in $q^\mu \rightarrow 0$ are



Thus,

$$\lim_{q^\mu \rightarrow 0} M_2(k, q) = 0.$$

$$\frac{i}{(k - q)^2 - m^2} = \frac{i}{-m^2 - 2k \cdot q + \mathcal{O}(q^2)},$$

5. Conclusion

- We consider the classical vacua of the $U(N)$ gauge model with partially broken $N=2$ supersymmetry and see that there are the $N=1$ supersymmetric vacua.
- We saw that gluino condensate effective superpotential is deformed
- We have also discussed the low energy theorem for the Nambu-Goldstone fermion.

N=2 supersymmetry

R transformation act on L:

$$R\mathcal{L}(\xi)R^{-1} = \mathcal{L}(-\xi)$$

$$\text{1st susy transformation: } \delta^{(1,\xi)}\mathcal{L}(\xi) = 0$$

This is ordinary N=1 susy transformation.

$$\text{2nd susy transformation: } \delta^{(2,\xi)}\mathcal{L}(\xi) = 0$$

The definition of $\delta^{(2,\xi)}$ is $\delta^{(2,\xi)} = R\delta^{(1,-\xi)}R^{-1}$

$$\begin{aligned}\text{Thus, } \delta^{(2,\xi)}\mathcal{L}(\xi) &= \left(R\delta^{(1,-\xi)}R^{-1}\right)\left(R\mathcal{L}(-\xi)R^{-1}\right) \\ &= R\left(\delta^{(1,-\xi)}\mathcal{L}(-\xi)\right)R^{-1} \\ &= 0\end{aligned}$$