Extra Dimensional Models with Magnetic Fluxes

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1. Introduction
2. Magnetized extra dimensions
3. Models
4. N-point couplings and flavor symmetries
5. Summary

based on

Abe, T.K., Ohki, arXiv: 0806.4748

Abe, Choi, T.K., Ohki, 0812.3534, 0903.3800, 0904.2631, 0907.5274,

Choi, T.K., Maruyama, Murata, Nakai, Ohki, Sakai, 0908.0395

1 Introduction

1-1. Introduction to higher D theory

Superstring theory predicts 10D space-time.

Extra dimensional field theories,

in particular

string-derived extra dimensional field theories,

play important roles in particle physics

as well as cosmology .

Extra dimensions

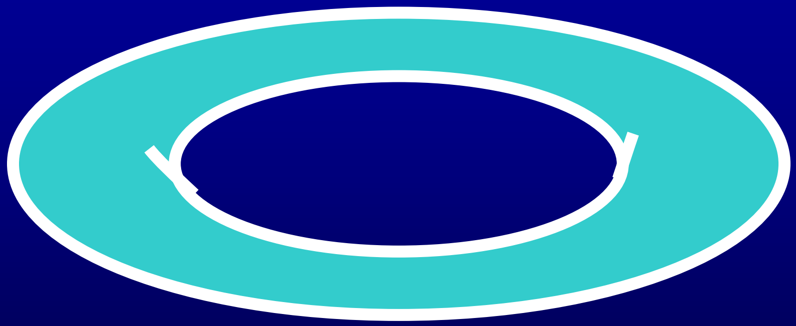
4 + n dimensions

4D ⇒ our 4D space-time

nD ⇒ compact space

Examples of compact space

torus

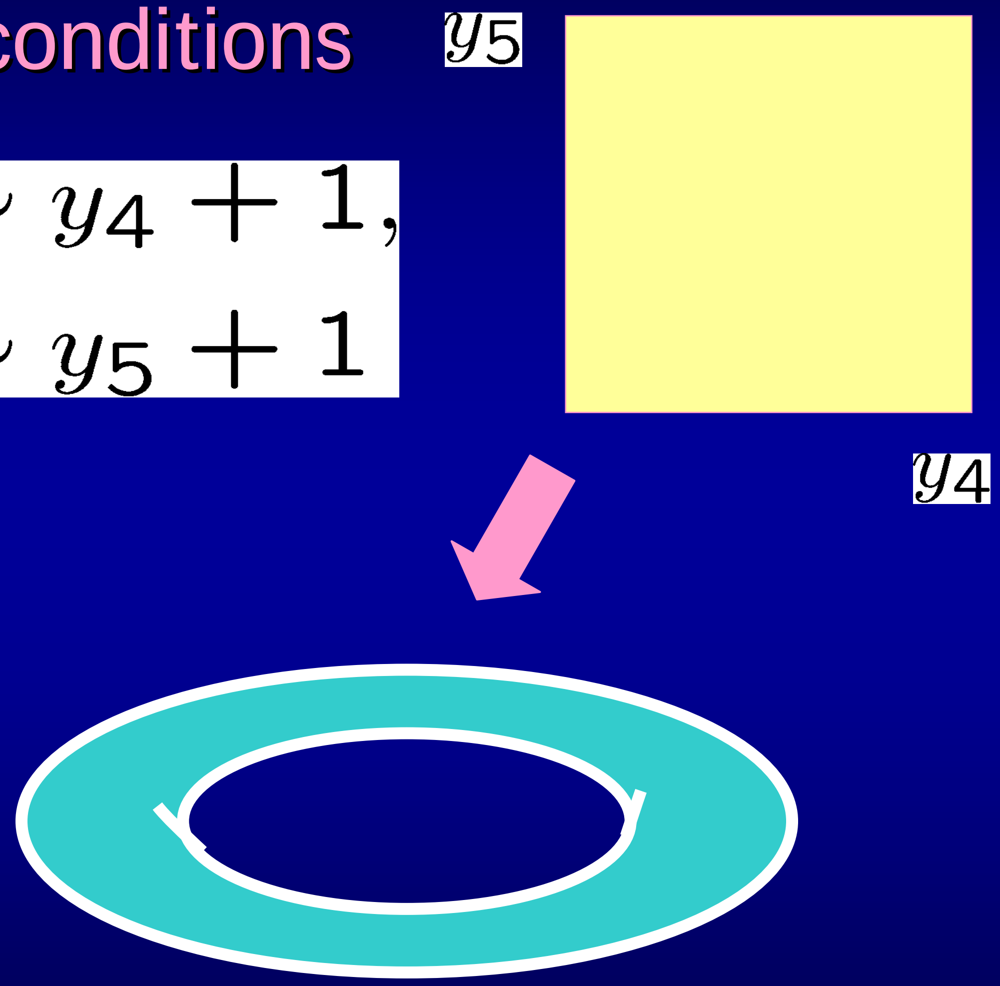


Torus

boundary conditions

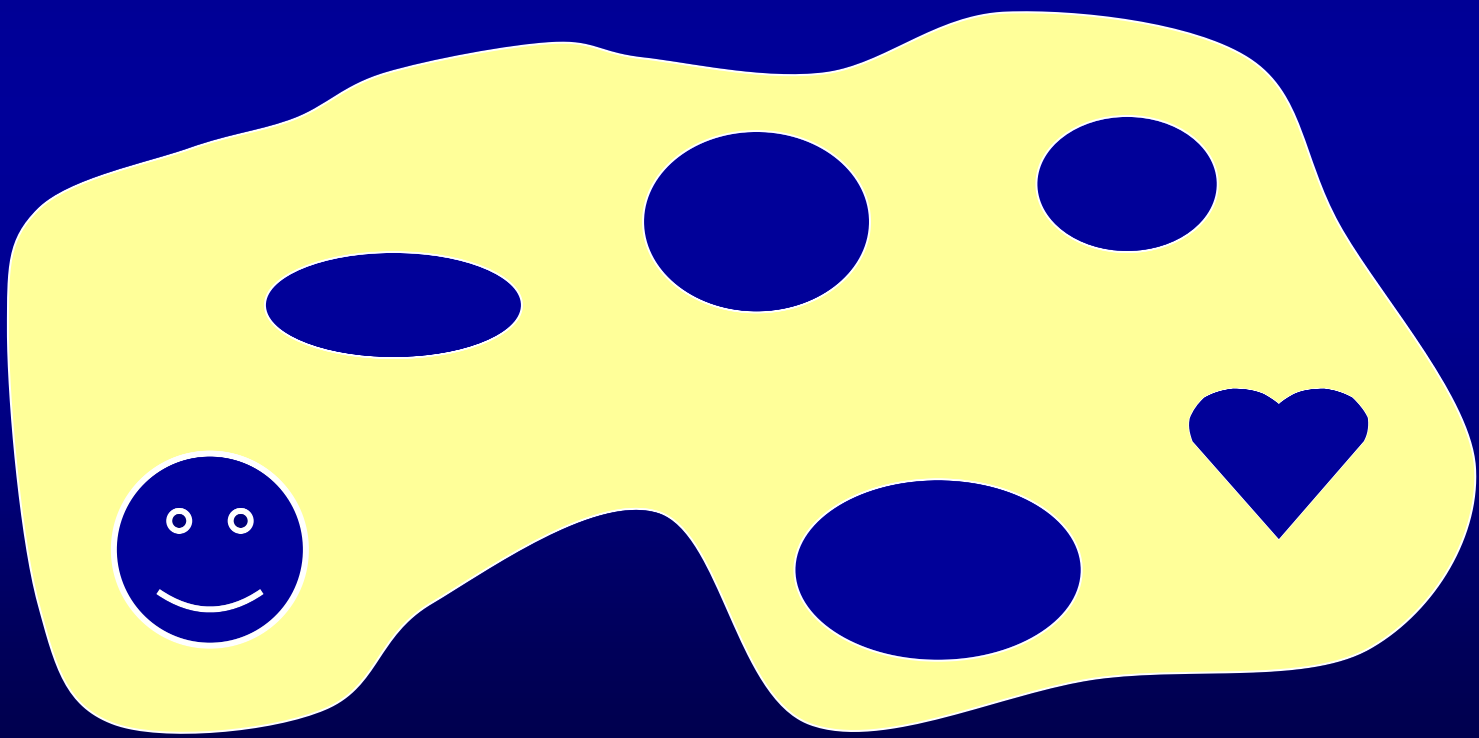
$$y_4 \sim y_4 + 1,$$
$$y_5 \sim y_5 + 1$$

torus



More complicated compact space

Calabi-Yau manifold, orbifold, etc.



Field theory in higher dimensions

10D ⇒ 4D our space-time + 6D space

10D vector

$$A_M \Rightarrow A_\mu , A_m$$

4D vector + 4D scalar

SO(10) spinor ⇒ SO(4) spinor

x SO(6) spinor

internal quantum number

Field theory in higher dimensions

Mode expansions

$$\Delta_{10} A_m = (\Delta_4 + \Delta_6) A_m = 0$$

$$i(\Gamma^\mu D_\mu + \Gamma^m D_m)\lambda = 0$$

$$\lambda(x^\mu, y^m) = \sum_n \chi_n(x^\mu) \times \psi_n(y_m),$$

$$A_M(x^\mu, y^m) = \sum_n \varphi_{n,M}(x^\mu) \times \phi_{n,M}(y_m)$$

$$i\Gamma_m D^m \psi_n(y) = m_n \psi_n,$$

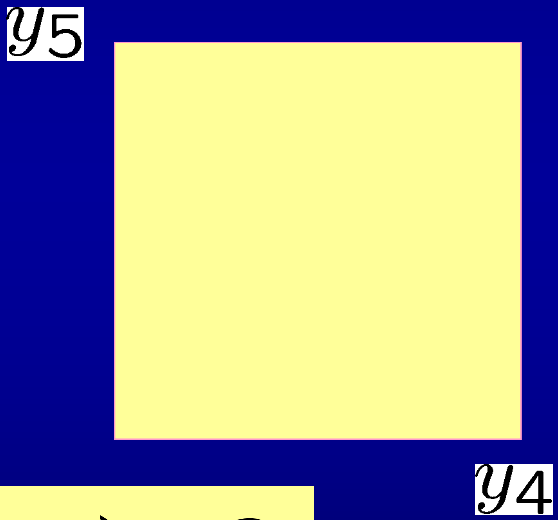
$$\Delta_6 \phi_{n,M}(y) = M^2_{n,M} \phi_{n,M}$$

We concentrate on zero-modes.

Zero-modes

We have to solve the zero-mode eq.

$$i\gamma^{m} D_{m}\psi = 0$$



Boundary conditons

$$y_4 \sim y_4 + 1,$$
$$y_5 \sim y_5 + 1$$

$$\psi(y_4 + 1, y_5) = \psi(y_4, y_5) \quad ?$$
$$\psi(y_4, y_5 + 1) = \psi(y_4, y_5) \quad ?$$

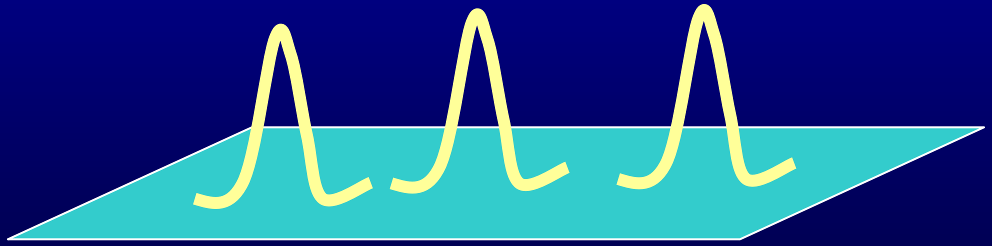
Zero-modes

Zero-mode equation

$$i\gamma^{m} D_{m} \psi = 0$$

⇒ non-trival zero-mode profile

the number of zero-modes



4D effective theory

Higher dimensional Lagrangian (e.g. 10D)

$$L_{10} = g\int d^4x d^6y\, \overline{\lambda}(x,y) A(x,y) \lambda(x,y)$$

integrate the compact space ⇒ 4D theory

$$L_4 = Y\int d^4x \overline{\chi}(x)\varphi(x)\chi(x)$$

$$Y = g\int d^6y \overline{\psi}(y)\phi(y)\psi(y)$$

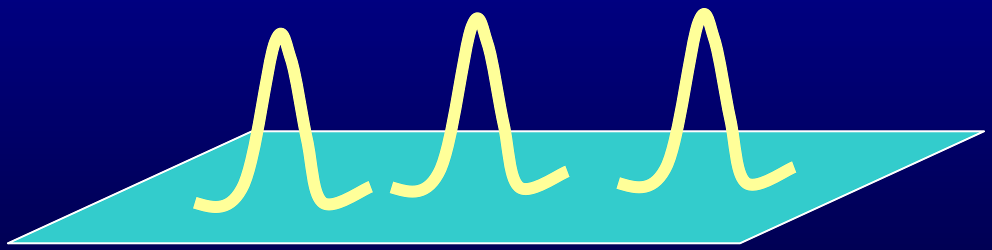
Coupling is obtained by the overlap integral of wavefunctions

Couplings in 4D

Zero-mode profiles are quasi-localized

far away from each other in compact space

⇒ suppressed couplings



1-2. Introduction to magnetized torus
Chiral theory

When we start with extra dimensional field theories,

how to realize chiral theories is one of important issues from the viewpoint of particle physics.

$$i\gamma^m D_m \psi = 0$$

Zero-modes between chiral and anti-chiral fields are different from each other on certain backgrounds, e.g. CY.

Torus with magnetic flux

$$i\gamma^m D_m \psi = 0$$

The limited number of solutions with non-trivial backgrounds are known.

Torus background with magnetic flux is one of interesting backgrounds, where one can solve zero-mode Dirac equation.

Magnetic flux

Indeed, several studies have been done in both extra dimensional field theories and string theories with magnetic flux background.

In particular, magnetized D-brane models are T-duals of intersecting D-brane models.

Several interesting models have been constructed in intersecting D-brane models, that is, the starting theory is U(N) SYM.

Magnetized D-brane models

The (generation) number of zero-modes
is determined by the size of magnetic flux.

Zero-mode profiles are quasi-localized.

=> several interesting phenomenology

Phenomenology of magnetized brane models

It is important to study phenomenological aspects of magnetized brane models such as

massless spectra from several gauge groups,

U(N), SO(N), E6, E7, E8, ...

Yukawa couplings and higher order n-point couplings in 4D effective theory,

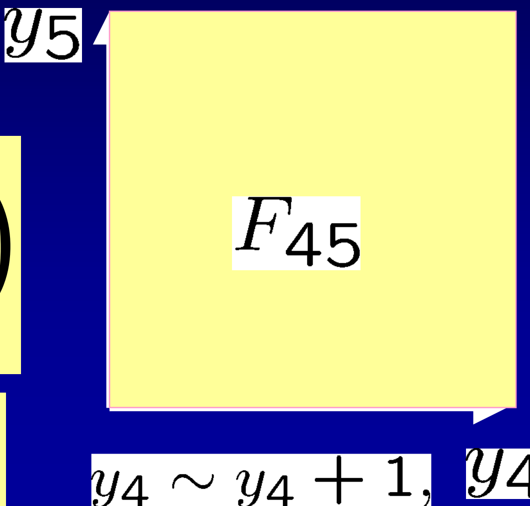
their symmetries like flavor symmetries,

Kahler metric, etc.

It is also important to extend such studies on torus background to other backgrounds with magnetic fluxes, e.g. orbifold backgrounds.

量子力学の復習：磁場中の粒子(Landau)　$U(1)$

$$F_{45} = 2\pi b,\ \ A_4 = 0,\ \ A_5 = 2\pi y_4$$



$$H = \frac{1}{2m}\left(P_4^2 + (P_5 - 2\pi b y_4)^2\right)$$

$$[H, P_5] = 0 \quad \Rightarrow \quad P_5 = 2\pi k$$

$y_4 \sim y_4 + 1,$

$y_5 \sim y_5 + 1$

$$H = \frac{1}{2m}\left(P_4^2 + 4\pi^2 b^2 (y_4 - k/b)^2\right)$$

座標がk/bずれた調和振動子　　　b=整数

b個の基底状態　　　k=0,1,2,…………,(b-1)

2. Extra dimensions with magnetic fluxes: basic tools

2-1. Magnetized torus model

We start with N=1 super Yang-Mills theory in D = 4+2n dimensions.

For example, 10D super YM theory consists of gauge bosons (10D vector) and adjoint fermions (10D spinor).

We consider 2n-dimensional torus compactification with magnetic flux background.

Higher Dimensional SYM theory with flux

Cremades, Ibanez, Marchesano, '04

4D Effective theory <= dimensional reduction

$$\mathcal{L}_{SYM} = -\frac{1}{4g^2} Tr\{F^{MN} F_{MN}\} + \frac{i}{2g^2} Tr\{\bar{\lambda} \Gamma^M D_M \lambda\}$$

$$\begin{aligned} \lambda(x^\mu, y^m) &= \sum_n \chi_n(x^\mu) \times \psi_n(y_m), \\ A_M(x^\mu, y^m) &= \sum_n \varphi_{n,M}(x^\mu) \times \phi_{n,M}(y_m) \end{aligned}$$

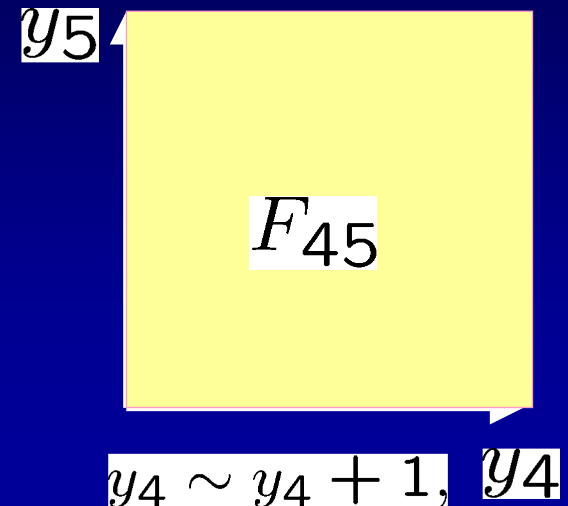
⇒

$$\begin{aligned} i\Gamma_m D^m \psi_n(y) &= m_n \psi_n, \\ \Delta_6 \phi_{n,M}(y) &= M_{n,M}^2 \phi_{n,M} \end{aligned}$$

The wave functions → eigenstates of corresponding internal Dirac/Laplace operator.

Higher Dimensional SYM theory with flux $U(1)$

Abelian gauge field on magnetized torus T^2



$$y_4 \sim y_4 + 1, \quad y_5 \sim y_5 + 1$$

Constant magnetic flux $F_{45} = b,$

gauge fields of background $\begin{cases} A_4 = 0, \\ A_5 = b y_4 \end{cases}$

The boundary conditions on torus (transformation under torus translations)

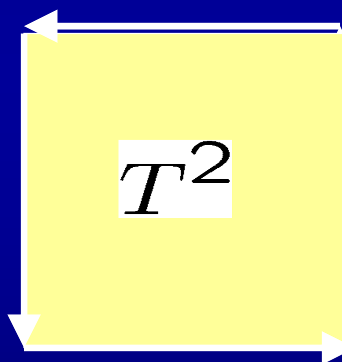
$$\begin{cases} A_m(y_4 + 1, y_5) = A_m(y_4, y_5) + \partial_m \chi_4, & \chi_4 = b y_5, \\ A_m(y_4, y_5 + 1) = A_m(y_4, y_5) + \partial_m \chi_5, & \chi_5 = 0, \end{cases}$$

Higher Dimensional SYM theory with flux $U(1)$

We now consider a complex field $\psi(y_4, y_5)$ with charge Q (+/-1)

$$\begin{cases} \psi(y_4+1, y_5) = e^{iQ\chi_4}\psi(y_4, y_5) = e^{iQby_5}\psi(y_4, y_5), \\ \psi(y_4, y_5+1) = e^{iQ\chi_5}\psi(y_4, y_5) = \psi(y_4, y_5), \end{cases}$$

Consistency of such transformations under a contractible loop in torus which implies Dirac's quantization conditions.



$$\frac{b}{2\pi} = M \in Z$$

Dirac equation

ψ is the two component spinor.

$$\psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}, \quad \Gamma^4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \Gamma^5 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\begin{cases} \left[\bar{\partial} + 2\pi M y_4\right]\psi_+(y) = 0 \\ \left[\partial - 2\pi M y_4\right]\psi_-(y) = 0 \end{cases}$$

with twisted boundary conditions (Q=1)

$$\begin{aligned} \psi(y_4+1, y_5) &= e^{2\pi i M y_5}\psi(y_4, y_5), \\ \psi(y_4, y_5+1) &= \psi(y_4, y_5), \end{aligned}$$

Dirac equation and chiral fermion

|M| independent zero mode solutions in Dirac equation.

$$\Theta^j(y_4, y_5) = N_j e^{-M\pi y_4^2} \cdot \vartheta \begin{bmatrix} j/M \\ 0 \end{bmatrix} (M(y_4 + iy_5), Mi)$$

$$(j = 0, 1, \cdots, |M| - 1)$$

$$\vartheta \begin{bmatrix} a \\ b \end{bmatrix} (\nu, \tau) \equiv \sum_n e^{\pi i (n+a)^2 \tau} e^{2\pi (a+n)(\nu+b)}$$

(Theta function)

Properties of theta functions

$$\vartheta \begin{bmatrix} a \\ b \end{bmatrix} (\nu + m, \tau) = e^{2\pi i m a} \cdot \vartheta \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\vartheta \begin{bmatrix} a \\ b \end{bmatrix} (\nu + m\tau, \tau) = e^{-\pi m^2 \tau - 2\pi i m (\nu + b)} \cdot \vartheta \begin{bmatrix} a \\ b \end{bmatrix}$$

chiral fermion

$M \gtrless 0 \Rightarrow$ $\psi_{+/-}$:Normalizable mode

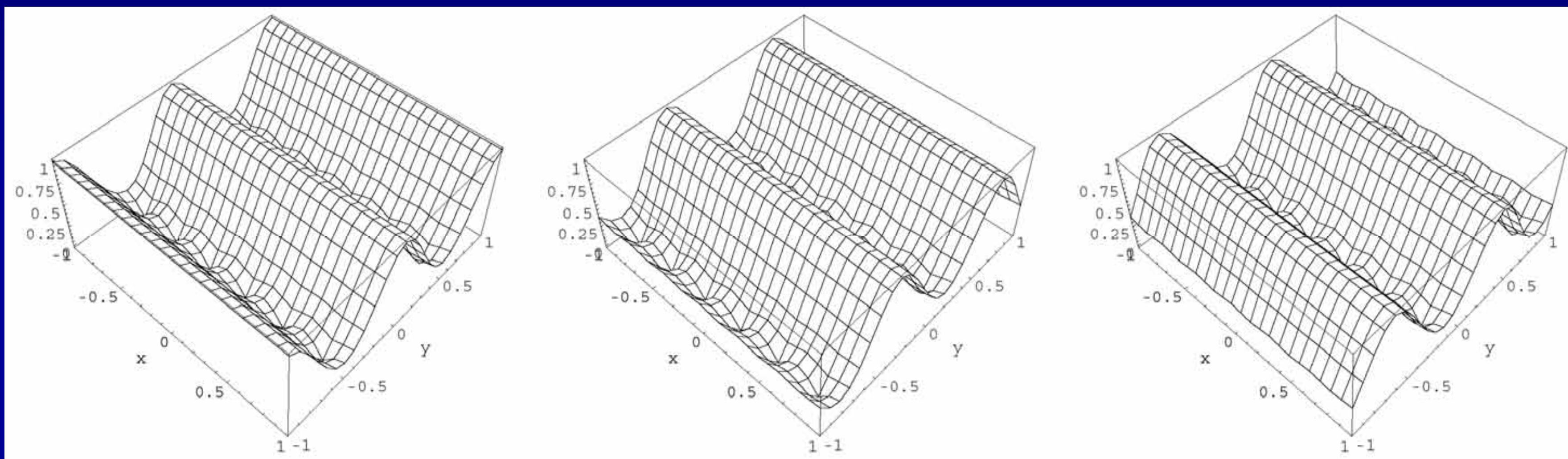
$\psi_{-/+}$:Non-normalizable mode

By introducing magnetic flux, we can obtain chiral theory.

Wave functions

For the case of M=3

$|\Theta^0(y)|$ $|\Theta^1(y)|$ $|\Theta^2(y)|$



Wave function profile on toroidal background

Zero-modes wave functions are quasi-localized far away each other in extra dimensions. Therefore the hierarchirally small Yukawa couplings may be obtained.

Fermions in bifundamentals $(N = N_a + N_b)$

$$F_{45} = 2\pi \begin{pmatrix} M_a \mathbf{1}_{N_a \times N_a} & 0 \\ 0 & M_b \mathbf{1}_{N_b \times N_b} \end{pmatrix}.$$

Breaking the gauge group $U(N) \to U(N_a) \times U(N_b)$

(Ablian flux case $M_a, M_b \in Z$)

The gaugino fields

$$\lambda(x,y) = \begin{pmatrix} \lambda^{aa}(x,y) & \lambda^{ab}(x,y) \\ \lambda^{ba}(x,y) & \lambda^{bb}(x,y) \end{pmatrix}.$$

⇒ λ^{aa} and λ^{bb} **gaugino of unbroken gauge** Adj N_a, Adj N_b.

λ^{ab} and λ^{ba} **bi-fundamental matter fields** $(N_a, \bar{N}_b), (\bar{N}_a, N_b)$.

Bi-fundamental

Gaugino fields in off-diagonal entries correspond to bi-fundamental matter fields and the difference M= m-m' of magnetic fluxes appears in their Dirac equation.

Zero-modes Dirac equations

$$\begin{pmatrix} \bar{\partial}\psi_+^{aa} & \left[\bar{\partial} + 2\pi(M_a - M_b)y_4\right]\psi_+^{ab} \\ \left[\bar{\partial} + 2\pi(M_b - M_a)y_4\right]\psi_+^{ba} & \bar{\partial}\psi_+^{bb} \end{pmatrix} = 0.$$

$$\begin{pmatrix} \partial\psi_-^{aa} & \left[\partial - 2\pi(M_a - M_b)y_4\right]\psi_-^{ab} \\ \left[\partial - 2\pi(M_b - M_a)y_4\right]\psi_-^{ba} & \partial\psi_-^{bb} \end{pmatrix} = 0.$$

No effect due to magnetic flux for adjoint matter fields, λ^{aa} and λ^{bb}

Total number of zero-modes of λ^{ab} ⇨ $I_{ab} = |M_a - M_b|.$

$M_a - M_b > 0 \Rightarrow$

ψ_+^{ab}, ψ_-^{ba} :Normalizable mode

ψ_-^{ab}, ψ_+^{ba} :Non-Normalizable mode

4D chiral theory

10D spinor

light-cone 8s $(\pm, \pm\ \pm\ \pm)$

even number of minus signs

1st ⇒ 4D, the other ⇒ 6D space

If all of λ^{ab} and λ^{ba} $(N_a, \bar{N}_b), (\bar{N}_a, N_b)$. appear

in 4D theory, that is non-chiral theory.

If $M_a - M_b > 0 \Rightarrow$ for all torus,

only $\lambda^{ab} \left(N_a, \overline{N_b}\right) (+, +\ +\ +)$

appear for 4D helicity fixed.

⇒ 4D chiral theory

2-2. Wilson lines

Cremades, Ibanez, Marchesano, '04,
Abe, Choi, T.K. Ohki, '09

torus without magnetic flux

constant Ai → mass shift

every modes massive

magnetic flux

$$\left[\bar{\partial} + 2\pi(My + a)\right]\psi_+ = 0$$
$$\left[\partial - 2\pi(My + a)\right]\psi_- = 0$$

the number of zero-modes is the same.

the profile: f(y) → f(y +a/M)

with proper b.c.

U(1)a*U(1)b theory

magnetic flux, Fa=2πM, Fb=0

Wilson line, Aa=0, Ab=C

matter fermions with U(1) charges, (Qa,Qb)

chiral spectrum,

for Qa=0, massive due to nonvanishing WL

when MQa >0, the number of zero-modes is MQa.

zero-mode profile is shifted depending on Qb,

$$f(z) \Rightarrow f(z + CQ_b/(MQ_a))$$

2-3. Magnetized orbifold models

We consider orbifold compactification
 with magnetic flux.

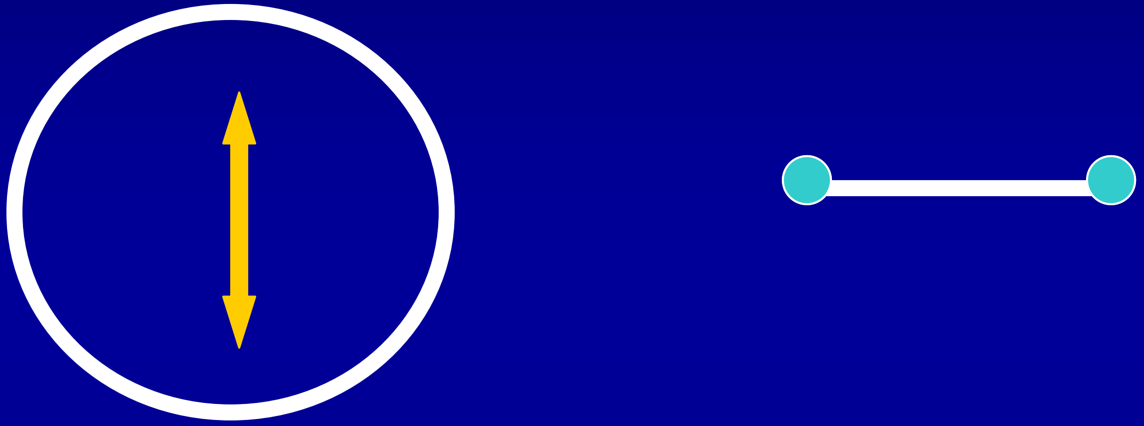
Orbifolding is another way to obtain chiral
 theory.

Magnetic flux is invariant under the Z2 twist.

We consider the Z2 and Z2xZ2' orbifolds.

Examples of orbifolds

S1/Z2 Orbifold



There are two singular points,
which are called fixed points.

Orbifold with magnetic flux

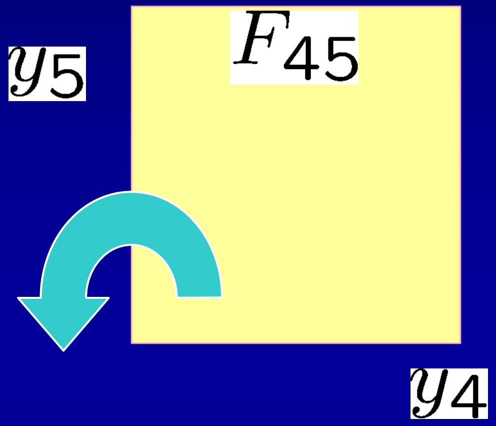
Abe, T.K., Ohki, '08

$$Z_2 : z (= y_4 + iy_5) \to -z$$

$$Z_2 : \psi_M^j(z) \to \psi_M^{M-j}(z)$$

$$\mathrm{Z}_2 \text{ even mode} : \psi_M^j + \psi_M^{M-j}$$

$$\mathrm{Z}_2 \text{ odd mode} : \psi_M^j - \psi_M^{M-j}$$



Note that there is no odd massless modes on the orbifold without magnetic flux.

Zero-modes

Even and/or odd modes are allowed as zero-modes on the orbifold with magnetic flux.

On the usual orbifold without magnetic flux, odd zero-modes correspond only to massive modes.

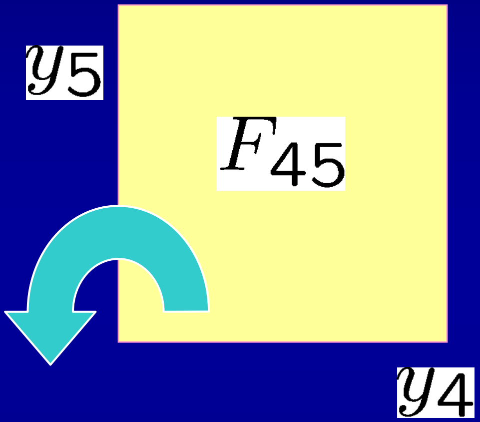
Adjoint matter fields are projected by orbifold projection.

Orbifold with magnetic flux

Abe, T.K., Ohki, '08

The number of even and odd zero-modes

$M = I^{ab}$	0	1	2	3	4	5	6	7	8	9	10
even	1	1	2	2	3	3	4	4	5	5	6
odd	0	0	0	1	1	2	2	3	3	4	4



We can also embed Z2 into the gauge space.

$$Z_2 : \psi(y_4, y_5) \rightarrow \psi(-y_4, -y_5) = (-i)\Gamma^4\Gamma^5 P\psi(-y_4, -y_5)$$

$$(P^2 = 1)$$

=> various models, various flavor structures

Localized modes on fixed points

We have degree of freedom to
introduce localized modes on fixed points
like quarks/leptons and higgs fields.

That would lead to richer flavor structure.

3. Models

We can construct several models by using the above model building tools.

What is the starting theory ?

10D SYM or 6D SYM (+ hyper multiplets),

gauge groups, U(N), SO(N), E6, E7,E8,...

What is the gauge background ?

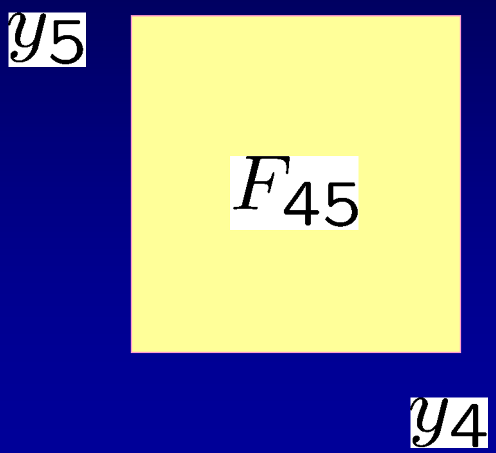
the form of magnetic fluxes, Wilson lines.

What is the geometrical background ?

torus, orbifold, etc.

U(N) theory on T6

$$F_{z\bar{z}} = 2\pi i \begin{pmatrix} m_1 \mathrm{I}_{N_1} & & 0 \\ & \ddots & \\ 0 & & m_k \mathrm{I}_{N_k} \end{pmatrix}$$



$$y_4 \sim y_4 + 1,$$
$$y_5 \sim y_5 + 1$$

$$\mathrm{I}_N : (N \times N) \quad \text{identity} \quad \text{matrix}$$

$$z = y_4 + i y_5$$

gauge group

$$U(N) \Rightarrow \prod_{i=1}^{k} U(N_i)$$

U(N) SYM theory on T6

$$F_{z\bar{z}} = 2\pi i \begin{pmatrix} m_1 \mathrm{I}_{N_1} & & 0 \\ & m_2 \mathrm{I}_{N_2} & \\ 0 & & m_3 \mathrm{I}_{N_3} \end{pmatrix}$$

$$N_1 = 4, N_2 = 2, N_3 = 2$$

$$U(4) \times U(2)_L \times U(2)_R$$

Pati-Salam group up to U(1) factors

$$(m_1 - m_2) = (m_3 - m_1) = 3 \text{ for the first } T^2$$
$$(m_1 - m_2) = (m_3 - m_1) = 1 \text{ for the other tori}$$

Three families of matter fields

with many Higgs fields

$$(4,2,1) + (\overline{4},1,2)$$

Orbifolding can lead to various 3-generation PS models.

See Abe, Choi, T.K., Ohki, '08

Wilson line breaking

$$U(4)\times U(2)_L\times U(2)_R$$

$\Rightarrow$

$$SU(3)\times SU(2)_L\times U(1)_Y$$

$$(4,2,1)_{\mathrm{L}} \Rightarrow Q + L$$

$$(\overline{4},1,2)_{\mathrm{R}} \Rightarrow u+d+e+\nu$$

Three families of quarks and leptons

E6 SYM theory on T6

Choi, et. al. '09

We introduce magnetix flux along U(1) direction,

which breaks E6 -> SO(10)*U(1)

$$78 = 45_0 + 1_0 + 16_1 + \overline{16}_{-1}$$

$$m_1 = 3, \; m_2 = 1, \; m_3 = 1$$

Three families of chiral matter fields 16

We introduce Wilson lines breaking

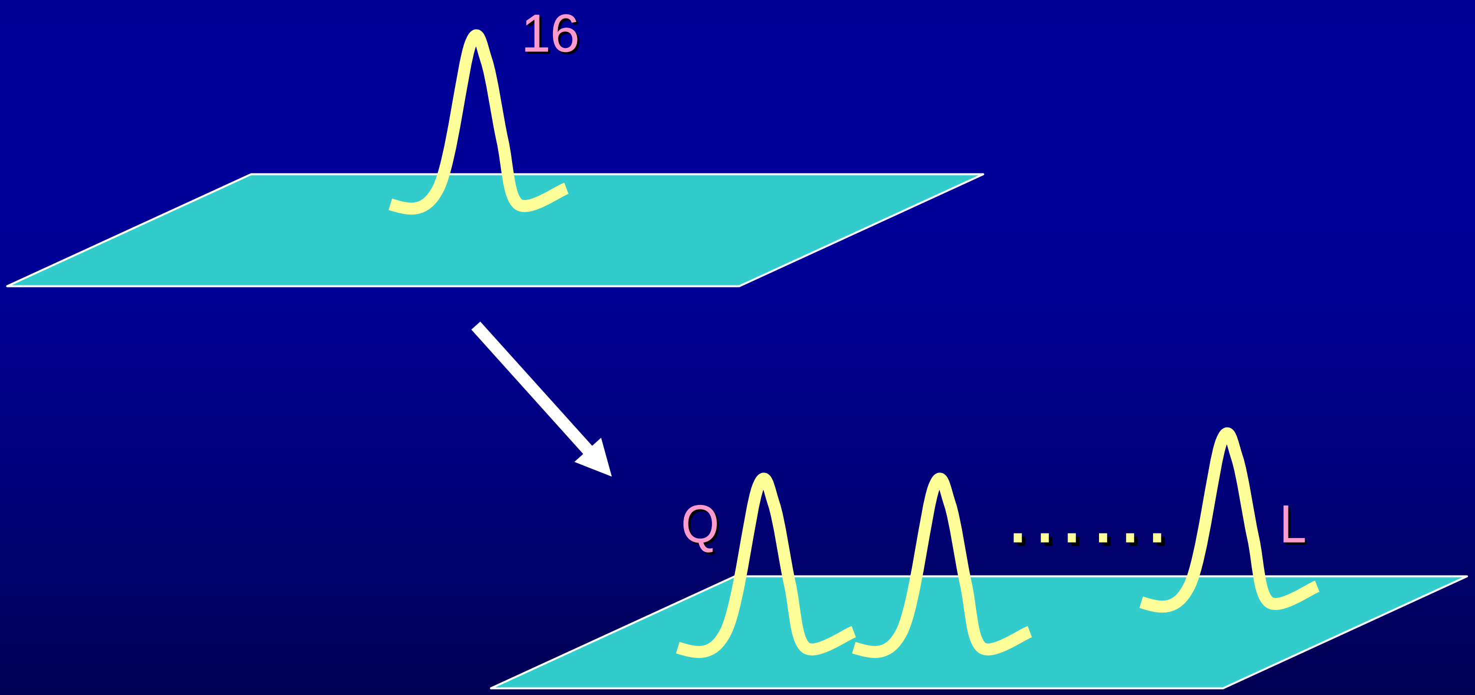
SO(10) -> SM group.

Three families of quarks and leptons matter fields

with no Higgs fields

Splitting zero-mode profiles

Wilson lines do not change the (generation) number of zero-modes, but change localization point.



E6 SYM theory on T6

There is no electro-weak Higgs fields

By orbifolding, we can derive a similar model with three generations of 16.

On the orbifold, there is singular points, i.e. fixed points.

We could assume consistently that electro-weak Higgs fields are localized modes on a fixed point.

E7, E8 SYM theory on T6

Choi, et. al. '09

E7 and E8 have more ranks (U(1) factors)

than E6 and SO(10).

Those adjoint rep. include various matter fields.

Then, we can obtain various models including

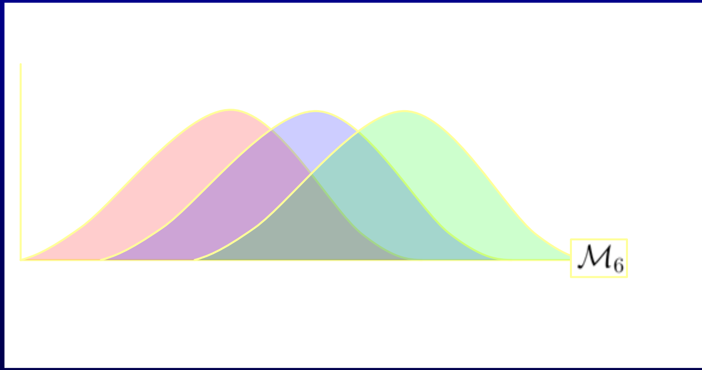
MSSM + vector-like matter fields

See for its detail our coming paper.

3. N-point couplings and flavor symmetries

The N-point couplings are obtained by overlap integral of their zero-mode w.f.'s.

$$Y = g\int d^2z\,\psi^i_M(z)\psi^j_N(z)\ldots\psi^k_P(z)$$



Zero-modes

Cremades, Ibanez, Marchesano, '04

$$\psi_M^j(z) = N_M \exp[i\pi M z \, \mathrm{Im}(z)] \cdot \vartheta \begin{bmatrix} j/M \\ 0 \end{bmatrix} (Mz, iM)$$

N_M : normalization factor, $\quad j = 1, \cdots, M$

Zero-mode w.f. = gaussian x theta-function

$$\psi_M^i(z) \cdot \psi_N^j(z) = \sum_{m=1}^{M+N} y_{ijm} \psi_{M+N}^{i+j+Mm}(z),$$

up to normalization factor

$$y_{ijm} = \vartheta \begin{bmatrix} (Ni - Mj + MNm)/(MN(M+N)) \\ 0 \end{bmatrix} (0, iMN(M+N))$$

3-point couplings

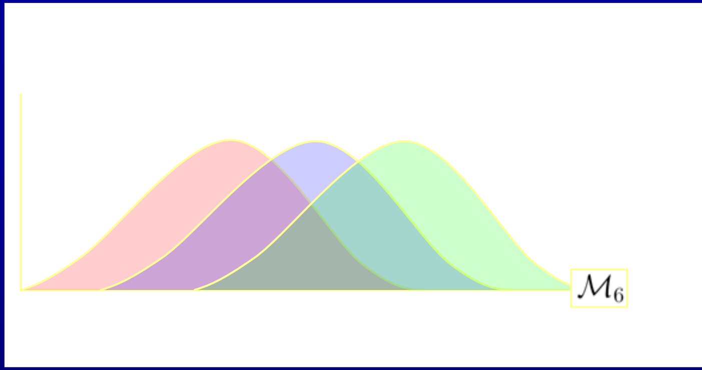
Cremades, Ibanez, Marchesano, ‘04

The 3-point couplings are obtained by overlap integral of three zero-mode w.f.’s.

$$Y_{ijk} = \int d^2 z\, \psi^i_M(z)\psi^j_N(z)\left(\psi^k_{M+N}(z)\right)^*$$

$$\int d^2 z\, \psi^i_M(z)\left(\psi^k_M(z)\right)^* = \delta^{ik}$$

$$Y_{ijk} = \sum_{m=1}^{M+N} \delta_{i+j+mM,k}\, y_{ijm}$$



up to normalization factor

Selection rule

$$\delta_{i+j+mM,k} \Rightarrow i+j+mM = k(M+N)$$

$$i+j=k \pmod{g} \quad \text{when} \quad g=\gcd(M,N)$$

Each zero-mode has a Zg charge,

which is conserved in 3-point couplings.

$$y_{ijm} = \vartheta\begin{bmatrix} (Ni - Mj + MNm)/(MN(M+N)) \\ 0 \end{bmatrix}(0, iMN(M+N))$$

up to normalization factor

4-point couplings

Abe, Choi, T.K., Ohki, '09

The 4-point couplings are obtained by overlap integral of four zero-mode w.f.'s.

$$Y_{ijkl} = \int d^2z\, \psi^i_M(z)\psi^j_N(z)\psi^k_P(z)\left(\psi^l_{M+N+P}(z)\right)^*$$

split

$$\int d^2z d^2z' \psi^i_M(z)\psi^j_N(z)\delta(z-z')\psi^k_P(z')\left(\psi^l_{M+N+P}(z')\right)^*$$

insert a complete set

$$\delta(z-z') = \sum_{\text{all modes}} \left(\psi^n_K(z)\right)^* \psi^n_K(z')$$

up to normalization factor

$$Y_{ijk\bar{l}} = \sum_s y_{ij\bar{s}}\, y_{sk\bar{l}}$$

for K=M+N

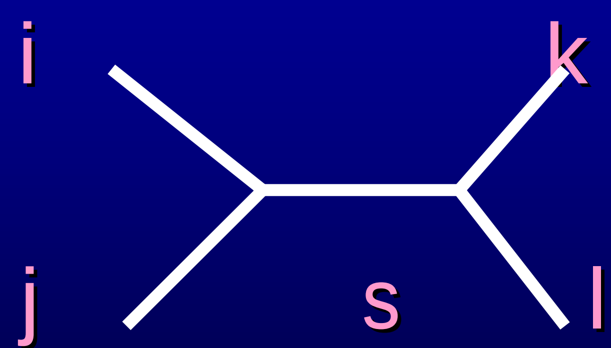
4-point couplings: another splitting

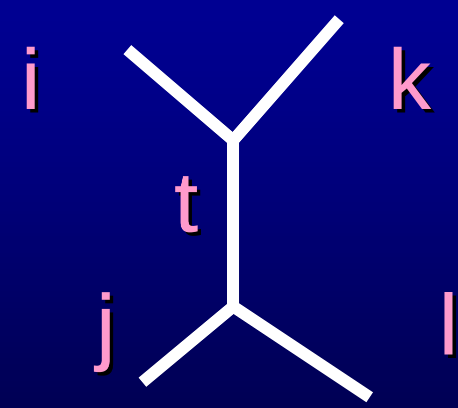
$$\int d^2z\, d^2z'\, \psi_M^i(z)\psi_P^k(z)\delta(z-z')\psi_N^j(z')\left(\psi_{M+N+P}^l(z')\right)^*$$

$$Y_{ijk\bar l} = \sum_t y_{ik\bar t}\, y_{tj\bar l}$$

$$Y_{ijk\bar l} = \sum_s y_{ij\bar s}\, y_{sk\bar l}$$

$$Y_{ijk\bar l} = \sum_t y_{ik\bar t}\, y_{tj\bar l}$$





N-point couplings

Abe, Choi, T.K., Ohki, '09

We can extend this analysis to generic n-point couplings.

N-point couplings = products of 3-point couplings
= products of theta-functions

This behavior is non-trivial. (It's like CFT.)
Such a behavior would be satisfied
not for generic w.f.'s, but for specific w.f.'s.

However, this behavior could be expected
from T-duality between magnetized
and intersecting D-brane models.

T-duality

The 3-point couplings coincide between magnetized and intersecting D-brane models.

explicit calculation

Cremades, Ibanez, Marchesano, '04

Such correspondence can be extended to 4-point and higher order couplings because of CFT-like behaviors, e.g.,

$$Y_{ijk\bar{l}} = \sum_s y_{ij\bar{s}} y_{sk\bar{l}}$$

Abe, Choi, T.K., Ohki, '09

Heterotic orbifold models

$$\left(\text{open string amplitude}\right)^2 = \left(\text{closed string amplitude}\right)$$

$$\begin{pmatrix} \text{couplings in} \\ \text{intersecting brane} \end{pmatrix}^2 = \begin{pmatrix} \text{coupling in} \\ \text{heterotic orbifold} \end{pmatrix}$$

Our results would be useful to n-point couplings of twsited sectors in heterotic orbifold models.

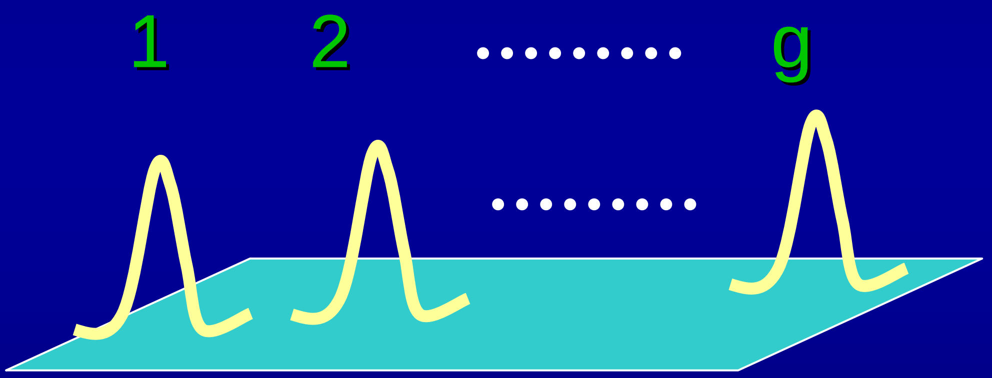
Twisted strings on fixed points might correspond to quasi-localized modes with magnetic flux,

zero modes profile = gaussian x theta-function

Non-Abelian discrete flavor symmetry

The coupling selection rule is controlled by
Zg charges.

For M=g,



Effective field theory also has a cyclic permutation symmetry of g zero-modes.

Non-Abelian discrete flavor symmetry

The total flavor symmetry corresponds to the closed algebra of

$$\begin{pmatrix} 1 & & & \\ & \rho & & \\ & & \ddots & \\ & & & \rho^{g-1} \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & \ddots & \ddots & 0 \\ 0 & \ddots & \ddots & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\rho = \exp[2\pi i / g]$$

That is the semidirect product of Zg x Zg and Zg.

For example,

g=2 D4

g=3 **Δ**(27)

Cf. heterotic orbifolds, T.K. Raby, Zhang, '04

T.K. Nilles, Ploger, Raby, Ratz, '06

Summary

We have studied phenomenological aspects of magnetized brane models.

Model building from U(N), E6, E7, E8

N-point couplings are comupted.
4D effective field theory has non-Abelian flavor symmetries, e.g. D4, $\Delta(27)$.

Orbifold background with magnetic flux is also important.